Beiträge zur
6. Diskussionssitzung der ITG-Fachgruppe

Angewandte Informationstheorie

Kaiserslautern, 07. Oktober 2005

Lehrstuhl für hochfrequente Signalübertragung und –verarbeitung
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LEHRSTUHL FÜR NACHRICHTENTECHNIK
Prof. Dr.-Ing. R. Urbansky

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Iterative Equalization in Fibre Optical Systems Using High-Rate RCPR, BCH and LDPC Codes

T. Schorr, A. Dittrich, W. Sauer-Greff, R. Urbansky
TU Kaiserslautern, Institute of Communications Engineering

6. Session
ITG-Fachgruppe "Angewandte Informationstheorie"
7. Oktober 2005, Kaiserslautern

Overview

- Digital Transmission System
- Channel EXIT chart analysis
- Code EXIT chart analysis
  - Convolutional Codes
  - Block Codes
  - LDPC Codes
- Conclusions

UNIVERSITY OF KAISERSLAUTERN

Communications Engineering
T. Schorr 2/2

Analytical Calculation of Thresholds for LDPC Codes transmitted over Binary Erasure Channels

Andac Dönmez, Thorsten Hehn and Johannes B. Huber

Insitute for Information Transmission

University of Erlangen-Nuremberg

Germany

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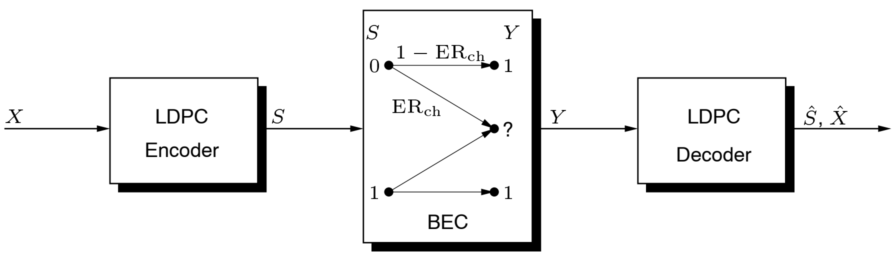
Introduction

- Setup: Transmission over *Binary Erasure Channel*



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- Successful decoding: Threshold dependent

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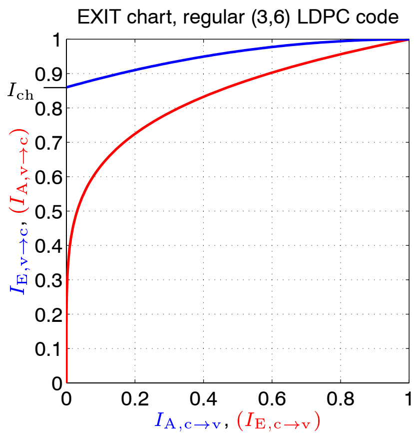
$$I_{\mathrm{ch}} = 1 - \mathrm{ER}_{\mathrm{ch}} = 0.86$$



$I_{\mathrm{A,c\to v}}$: A-priori info for variable nodes, $(I_{\mathrm{E,c\to v}})$: Extrinsic info from check nodes

Introduction

- Setup: Transmission over *Binary Erasure Channel*
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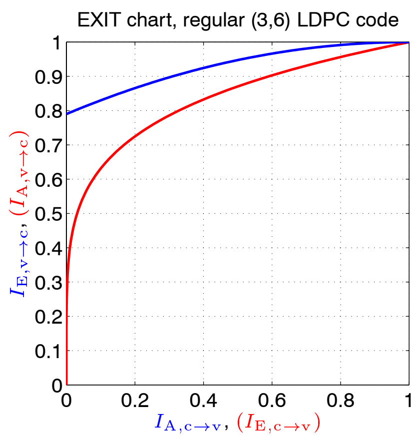
$$I_{\mathrm{ch}} = 1 - \mathrm{ER}_{\mathrm{ch}} = 0.79$$



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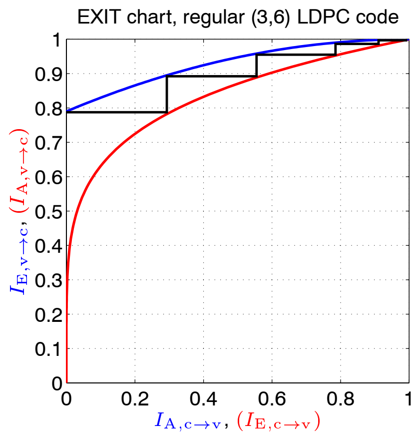

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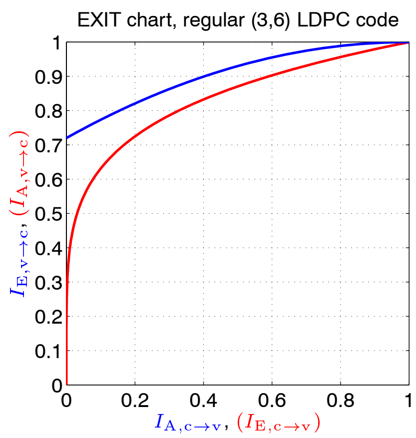
$$I_{\mathrm{ch}} = 1 - \mathrm{ER}_{\mathrm{ch}} = 0.72$$



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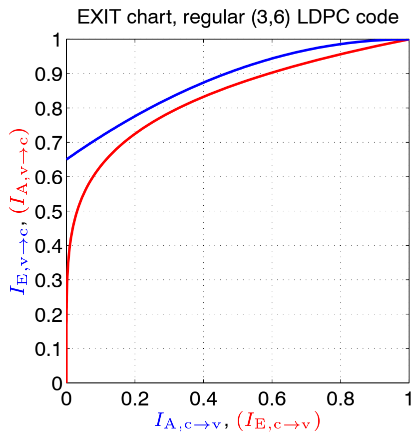

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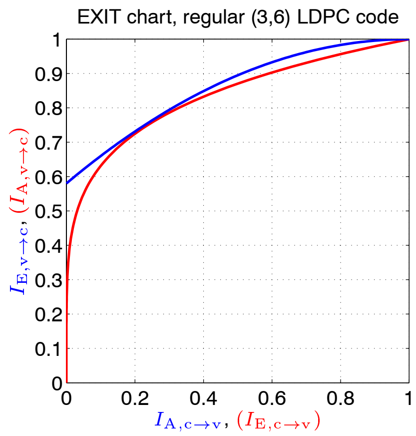
$$I_{\mathrm{ch}} = 1 - \mathrm{ER}_{\mathrm{ch}} = 0.58$$



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- Setup: Transmission over *Binary Erasure Channel*
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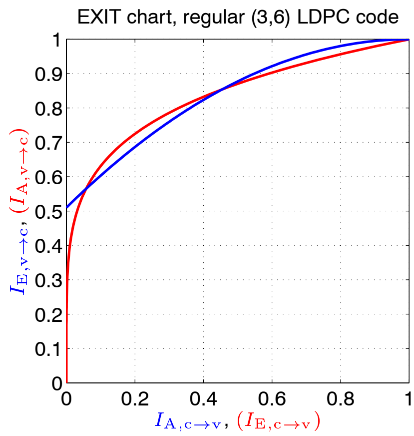
$$I_{\mathrm{ch}} = 1 - \mathrm{ER}_{\mathrm{ch}} = 0.51$$



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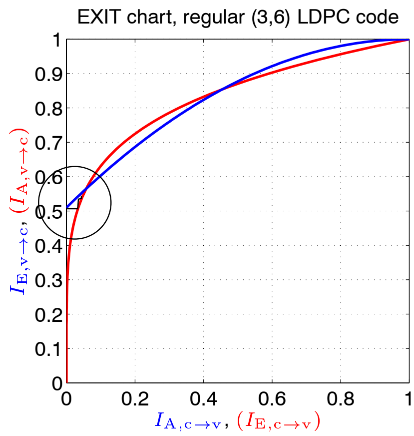

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- Setup: Transmission over *Binary Erasure Channel*
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- Visualization: EXIT charts

- Calculation of thresholds: *Density Evolution*, certain assumptions

Introduction

- Setup: Transmission over *Binary Erasure Channel*
- Successful decoding: Threshold dependent
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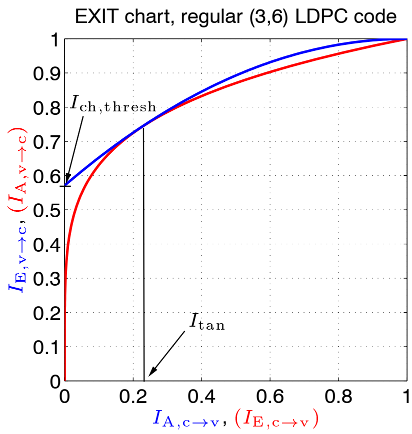
- Calculation of thresholds: *Density Evolution*, certain assumptions
- EXIT charts are exact for the binary erasure channel

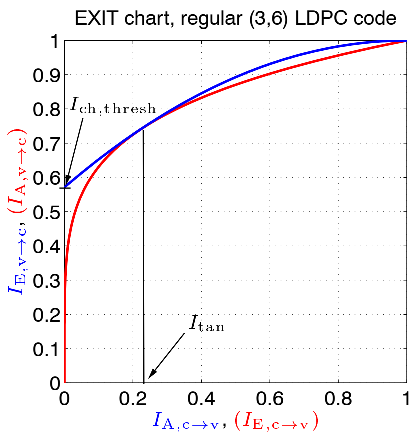
Calculate threshold analytically using EXIT charts?

Project



- Find channel information threshold $I_{\mathrm{ch,thresh}}$ and I_{tan} analytically

Project



- Find channel information threshold $I_{\mathrm{ch,thresh}}$ and I_{tan} analytically
- EXIT chart curves have to be tangent

Proceeding

- Denote abscissa of EXIT charts as I for convenience
- Define the variable-node function as $f_1(I, I_{\text{ch}})$:

$$f_1(I, I_{\text{ch}}) = 1 - (1 - I_{\text{ch}}) \cdot \lambda(1 - I)$$

- Define the check-node function as $f_2^{-1}(I)$

$$f_2^{-1}(I) = \rho(I)$$

- Define the inverted check-node function as $f_2(I)$

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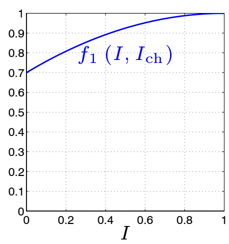

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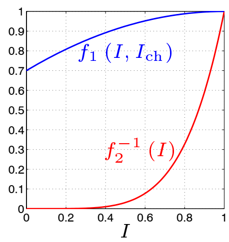

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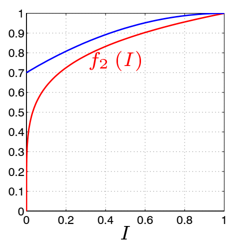

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- $\lambda(x)$, $\rho(x)$: Degree distributions from edge perspective with maximum degrees L and R

$$\lambda(x) = \sum_{i=2}^{L} \lambda_i x^{i-1}, \; \rho(x) = \sum_{i=2}^{R} \rho_i x^{i-1}$$

Proceeding

- Equate functions $f_1(I, I_{\mathrm{ch}})$ and $f_2(I)$
- Equate derivatives of $f_1(I, I_{\mathrm{ch}})$ and $f_2(I)$ w.r.t. I

$$f_2(I) = \rho^{-1}(I) = 1 - (1 - I_{\mathrm{ch}}) \cdot \lambda(1 - I) = f_1(I, I_{\mathrm{ch}})$$

$$\frac{\partial}{\partial I} f_2(I) = \frac{\partial}{\partial I} \rho^{-1}(I) = \frac{\partial}{\partial I}\Big(1 - (1 - I_{\mathrm{ch}}) \cdot \lambda(1 - I)\Big) = \frac{\partial}{\partial I} f_1(I, I_{\mathrm{ch}})$$

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This leads to the key equations

$$0 = \rho^{-1}(I)\widetilde{\lambda}(1 - I) + \lambda(1 - I)\widetilde{\rho^{-1}}(I) - \widetilde{\lambda}(1 - I)$$

$$\widetilde{\rho^{-1}}(I) = (1 - I_{\text{ch}})\widetilde{\lambda}(1 - I)$$

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$$0 = \rho^{-1}(I)\widetilde{\lambda}(1 - I) + \lambda(1 - I)\widetilde{\rho^{-1}}(I) - \widetilde{\lambda}(1 - I)$$

$$\widetilde{\rho^{-1}}(I) = (1 - I_{\text{ch}})\widetilde{\lambda}(1 - I)$$

with the defintions:

- $\widetilde{\rho^{-1}}(I) := \frac{\partial}{\partial I}\rho^{-1}(I)$
- $\widetilde{\lambda}(1 - I) := \frac{\partial}{\partial(1-I)}\lambda(1 - I) = \sum_{i=2}^{L}(i - 1) \cdot \lambda_i \cdot I^{i-2}$

Proceeding

- Equate functions $f_1(I, I_{\text{ch}})$ and $f_2(I)$
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- Solving first key equation: I_{tan}
- I_{tan} in second key equation: $I_{\text{ch,thresh}} = 1 - \left.\frac{\widetilde{\rho^{-1}}(I)}{\widetilde{\lambda}(1-I)}\right|_{I=I_{\text{tan}}}$

Proceeding: Regular case

For regular codes, the key equations become very simple:

$$0 = [(d_{\text{c}} - 1)(d_{\text{v}} - 1) - 1]I^{\frac{1}{d_{\text{c}}-1}} + I^{\frac{2-d_{\text{c}}}{d_{\text{c}}-1}} - (d_{\text{c}} - 1)(d_{\text{v}} - 1)$$
$$\frac{1}{d_{\text{c}} - 1} I^{\frac{2-d_{\text{c}}}{d_{\text{c}}-1}} = (1 - I_{\text{ch}})(d_{\text{v}} - 1)(1 - I)^{d_{\text{v}}-2}$$

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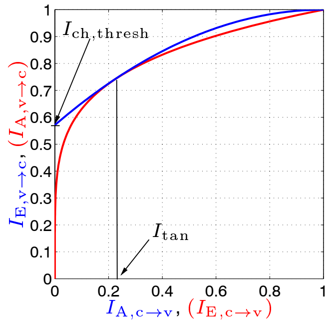
Example: For the regular $(3, 6)$ code we calculate:



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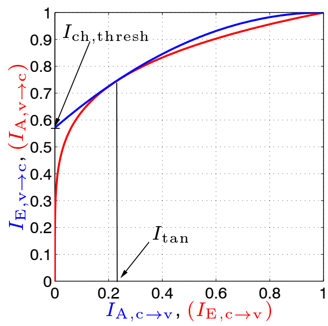

$$\begin{aligned} I_\mathrm{tan} &= 0.22105 \\ I_\mathrm{ch,thresh} &= 0.57056 \end{aligned}$$

Uniqueness of calculated threshold

Claim:

If variable-node and check-node degrees are larger than two, then $I_{\text{ch,thresh}}$ and $I_{\tan}$ are unique.

Sketch of Proof :

- $f_1\left(I, I_{\text{ch}}\right)$ and $f_2(I)$ are convex-$\cap$

Uniqueness of calculated threshold

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Sketch of Proof :

- $f_1\left(I, I_{\text{ch}}\right)$ and $f_2(I)$ are convex-$\cap$
- $f_1\left(I, I_{\text{ch}}\right)$ is convex-$\cap$ and $f_2^{-1}(I)$ is convex-$\cup$

Uniqueness of calculated threshold

Claim:

If variable-node and check-node degrees are larger than two, then $I_{\mathrm{ch,thresh}}$ and $I_{\tan}$ are unique.

Sketch of Proof :

- $f_1\left(I, I_{\mathrm{ch}}\right)$ and $f_2(I)$ are convex-$\cap$
- $f_1\left(I, I_{\mathrm{ch}}\right)$ is convex-$\cap$ and $f_2^{-1}(I)$ is convex-$\cup$
- Calculation of first and second derivative w.r.t. I is easy for $f_1\left(I, I_{\mathrm{ch}}\right)$ and $f_2^{-1}(I)$

Comparison to Density Evolution

- Density Evolution for the binary erasure channel:

$$\mathrm{ER}_l = \mathrm{ER}_{\mathrm{ch}} \cdot \lambda\left(1-\rho\left(1-\mathrm{ER}_{l-1}\right)\right),\ l \geq 1,\ \mathrm{ER}_0 = \mathrm{ER}_{\mathrm{ch}}$$

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- Requires a numerical search algorithm and depends on number of iterations

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- Requires a numerical search algorithm and depends on number of iterations

Example: Regular (3,6) code, binary search, 10^3 iterations

Code	Rate	Exact Thr. ($I_{\mathrm{ch,thresh}}$)	Thr. Density Evolution ($I_{\mathrm{ch,thresh}}$)
$(3,6)$	$1/2$	0.57056018558051	0.57056952861516

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- Requires a numerical search algorithm and depends on number of iterations

Example: Regular (3,6) code, binary search, 10^5 iterations

Code	Rate	Exact Thr. $(I_{\mathrm{ch,thresh}})$	Thr. Density Evolution $(I_{\mathrm{ch,thresh}})$
$(3,6)$	$1/2$	0.57056018558051	0.57056018651248

Comparison to Density Evolution

- Density Evolution for the binary erasure channel:

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- Requires a numerical search algorithm and depends on number of iterations

Example: Regular (3,6) code, binary search, 10^6 iterations

Code	Rate	Exact Thr. $(I_{\mathrm{ch,thresh}})$	Thr. Density Evolution $(I_{\mathrm{ch,thresh}})$
$(3,6)$	$1/2$	0.57056018558051	0.57056018558983

Comparison to Density Evolution

- Density Evolution for the binary erasure channel: Convergence requires the following condition which can be used for numerical threshold calculation

$$\mathrm{ER_{ch}} \cdot \lambda\left(1-\rho\left(1-\mathrm{x}\right)\right) < \mathrm{x}, \quad 0 < \mathrm{x} < \mathrm{ER_{ch}}$$

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- Convergence in EXIT chart method requires the following condition:

$$\rho\left(1-\left(1-\mathrm{I_{ch}}\right)\lambda\left(1-\mathrm{I}\right)\right) > \mathrm{I}, \quad \mathrm{I_{ch}} < \mathrm{I} < 1$$

Comparison to Density Evolution

- Density Evolution for the binary erasure channel: Convergence requires the following condition which can be used for numerical threshold calculation

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- Going from first inequality to second is possible with change of variables : EXIT chart method verifies the density evolution

Comparison to Density Evolution

- Start eith the first inequality and $u = 1 - x$

$$\begin{aligned} ER_{ch} \cdot \lambda\left(1-\rho\left(u\right)\right) &< 1-u \\ 1 - ER_{ch} \cdot \lambda\left(1-\rho\left(u\right)\right) &> u \end{aligned}$$

Comparison to Density Evolution

- Start eith the first inequality and $u = 1 - x$

$$\begin{aligned} ER_{ch} \cdot \lambda\left(1 - \rho\left(u\right)\right) &< 1 - u \\ 1 - ER_{ch} \cdot \lambda\left(1 - \rho\left(u\right)\right) &> u \end{aligned}$$

- say $v = 1 - \rho(u)$

$$\begin{aligned} 1 - ER_{ch} \cdot \lambda\left(v\right) &> \rho^{-1}\left(1 - v\right) \\ \rho\left(1 - ER_{ch} \cdot \lambda\left(v\right)\right) &> 1 - v \end{aligned}$$

Comparison to Density Evolution

- Start eith the first inequality and $u = 1 - x$

$$\begin{aligned} ER_{ch} \cdot \lambda\left(1 - \rho\left(u\right)\right) &< 1 - u \\ 1 - ER_{ch} \cdot \lambda\left(1 - \rho\left(u\right)\right) &> u \end{aligned}$$

- say $v = 1 - \rho(u)$

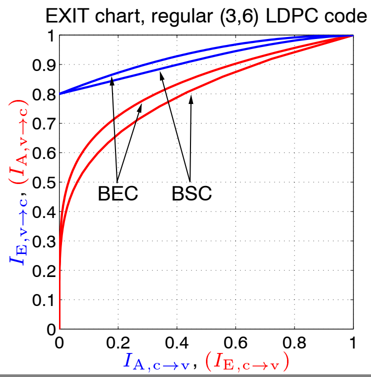
$$\begin{aligned} 1 - ER_{ch} \cdot \lambda\left(v\right) &> \rho^{-1}\left(1 - v\right) \\ \rho\left(1 - ER_{ch} \cdot \lambda\left(v\right)\right) &> 1 - v \end{aligned}$$

- Second inequality is obtained when $I = 1 - v$ and $ER_{ch} = 1 - I_{ch}$

$$\rho\left(1 - \left(1 - I_{ch}\right) \cdot \lambda\left(1 - I\right)\right) > I$$

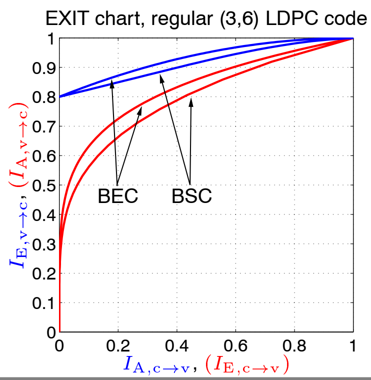
Application

- Extreme channels (from information combining point of view): BEC and BSC



Application

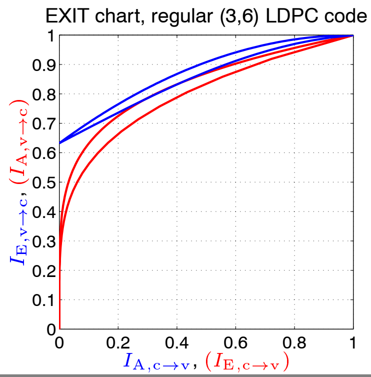
- Extreme channels (from information combining point of view): BEC and BSC
- Variable-node and check-node curve very close to each other



A. Dönmez, T. Hehn and J. B. Huber: Analytical Calculation of Thresholds for LDPC Codes transmitted over Binary Erasure Channels 11/12

Application

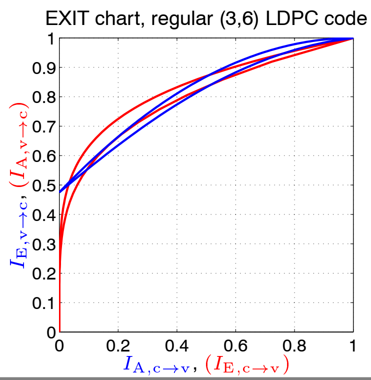
- Extreme channels (from information combining point of view): BEC and BSC
- Variable-node and check-node curve very close to each other
- Maximum and minimum threshold (mixed-channel assumption) close to each other



variable nodes: BSC
check nodes: BEC

Application

- Extreme channels (from information combining point of view): BEC and BSC
- Variable-node and check-node curve very close to each other
- Maximum and minimum threshold (mixed-channel assumption) close to each other



variable nodes: BEC
check nodes: BSC

Application

- Extreme channels (from information combining point of view): BEC and BSC
- Variable-node and check-node curve very close to each other
- Maximum and minimum threshold (mixed-channel assumption) close to each other

- Calculated value lies in between these bounds
- Good rule of thumb for any symmetric channel

Conclusions

- Analytical calculation of threshold for LDPC Codes transmitted over the binary erasure channel
- Independent of number of iterations or the accuracy of a search algorithm
- Verifies Density Evolution

Conclusions

- Analytical calculation of threshold for LDPC Codes transmitted over the binary erasure channel
- Independent of number of iterations or the accuracy of a search algorithm
- Verifies Density Evolution

- Good rule of thumb for threshold when transmitting over any arbitrary symmetric channel

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HS

Dynamic Resource Allocation in Future OFDM Based Mobile Radio Systems

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outline

HS

- the JOINT concept
- reference scenario
- dynamic resource allocation
- simulation results
- summary

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the B3G JOINT concept

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- key features:
  - OFDM based data transmission
  - TDD
  - service-area (SA) based structure
  - joint transmission (JT) & joint detection (JD)

JOINT - Joint Transmission and Detection Integrated Network

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conventional cellular vs. SA based network

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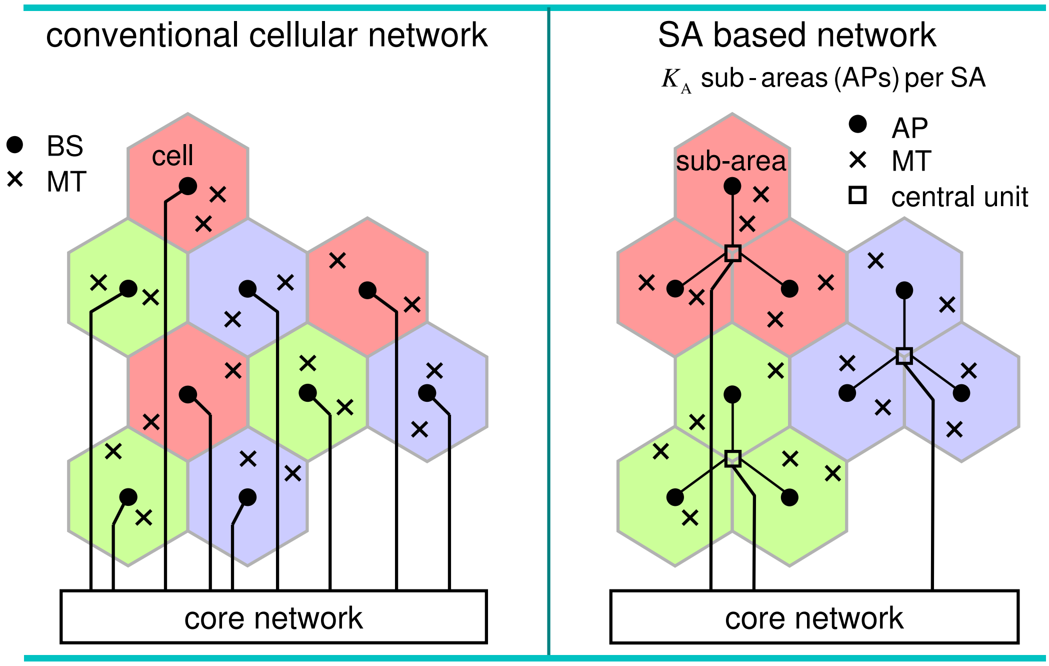

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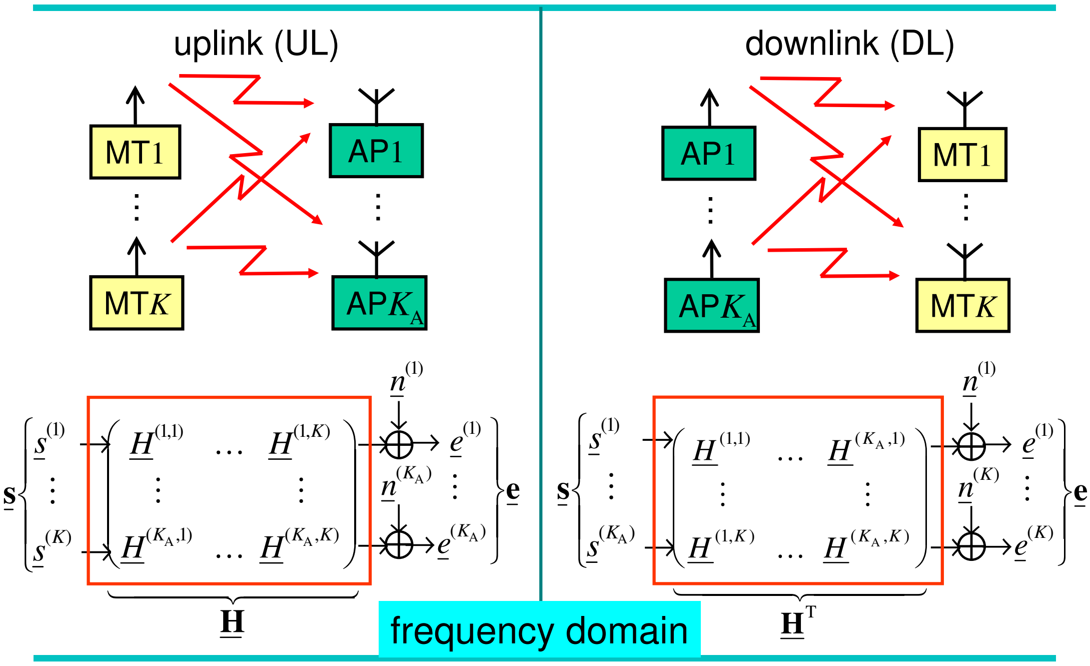
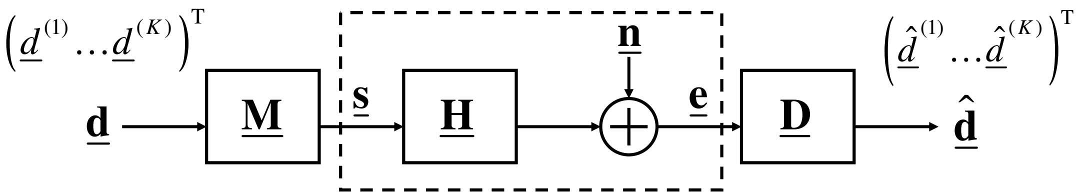
single subcarrier transmission model



linear JT and JD



interference free: $\underline{\mathbf{D}} \cdot \underline{\mathbf{H}} \cdot \underline{\mathbf{M}} = \mathbf{I} \rightarrow \hat{\underline{\mathbf{d}}} = \underline{\mathbf{d}} + \underline{\mathbf{D}} \cdot \underline{\mathbf{n}}$

(possible if $K_A \geq K$)

	$\underline{\mathbf{M}}$	$\underline{\mathbf{H}}$	$\underline{\mathbf{D}}$
UL: transmitter oriented (JD)	$\underline{\mathbf{M}} = \mathbf{I}$	known (JCE)	$(\underline{\mathbf{H}}^{*\mathrm{T}}\underline{\mathbf{H}})^{-1}\underline{\mathbf{H}}^{*\mathrm{T}}$
DL: receiver oriented (JT)	$\underline{\mathbf{H}}^{*\mathrm{T}}(\underline{\mathbf{H}}\,\underline{\mathbf{H}}^{*\mathrm{T}})^{-1}$	known (channel reciprocity)	$\underline{\mathbf{D}} = \mathbf{I}$

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reference multi-SA scenario

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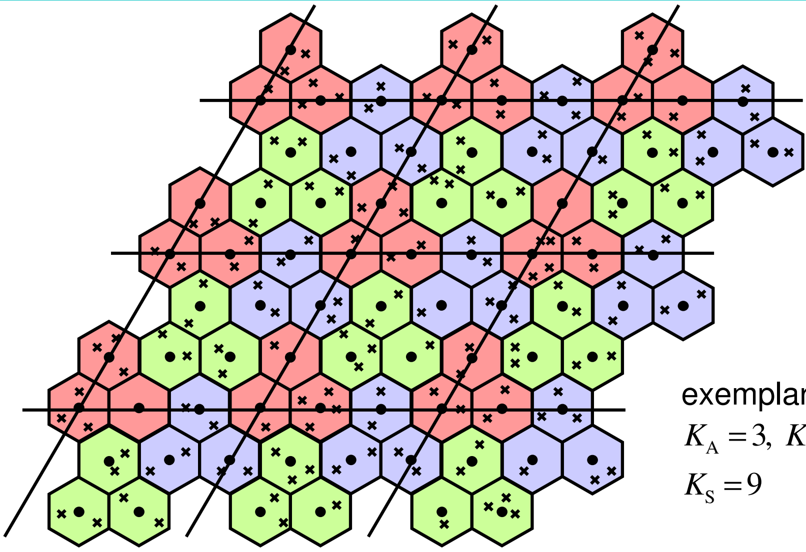

exemplary scenario:
$K_A = 3,\ K_M = 6,\ r = 3$
$K_S = 9$

K_A APs (sub-areas) per SA $\quad K_A = 1 \rightarrow$ cellular system

K_M MTs per SA $\quad$ reuse factor r $\quad K_S$ co-channel SAs

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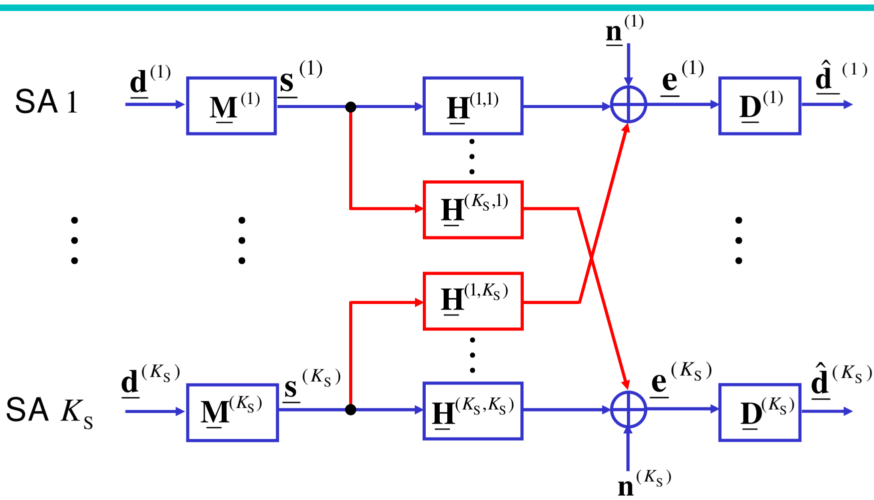
multi-SA transmission model

HS



$$\hat{\underline{\mathbf{d}}}^{(i)} = \underline{\mathbf{d}}^{(i)} + \underbrace{\underline{\mathbf{D}}^{(i)} \sum_{\substack{j=1 \\ j \neq i}}^{K_S} \underline{\mathbf{H}}^{(i,j)} \underline{\mathbf{M}}^{(j)} \underline{\mathbf{d}}^{(j)}}_{\text{inter-SA MAI}} + \underline{\mathbf{D}}^{(i)} \underline{\mathbf{n}}^{(i)}, \quad i = 1 \ldots K_S$$

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UL capacity

HS

$$\underline{\mathbf{M}}^{(i)} = \mathbf{I}$$

$$\underline{\mathbf{D}}^{(i)} = \left(\underline{\mathbf{H}}^{(i,i)*\mathrm{T}}\underline{\mathbf{H}}^{(i,i)}\right)^{-1}\underline{\mathbf{H}}^{(i,i)*\mathrm{T}}$$

$$\underline{\hat{\mathbf{d}}}^{(i)} = \underline{\mathbf{d}}^{(i)} + \sum_{\substack{j=1\\ j\neq i}}^{K_\mathrm{S}} \underbrace{\underline{\mathbf{D}}^{(i)}\underline{\mathbf{H}}^{(i,j)}}_{\underline{\mathbf{X}}^{(i,j)}}\underline{\mathbf{d}}^{(j)} + \underline{\mathbf{D}}^{(i)}\underline{\mathbf{n}}^{(i)}$$

UL capacity of the k-th user in SA $i,\ k=1\ldots K,\ i=1\ldots K_\mathrm{S}$

$$\boxed{C_k^{(i)} = \log_2\left(1+\frac{E_\mathrm{d}}{E_\mathrm{d}\sum_{\substack{j=1\\ j\neq i}}^{K_\mathrm{S}}\left[\underline{\mathbf{X}}^{(i,j)}\underline{\mathbf{X}}^{(i,j)*\mathrm{T}}\right]_{k,k} + \sigma^2\left[\left(\underline{\mathbf{H}}^{(i,i)*\mathrm{T}}\underline{\mathbf{H}}^{(i,i)}\right)^{-1}\right]_{k,k}}\right)}$$

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resource pool

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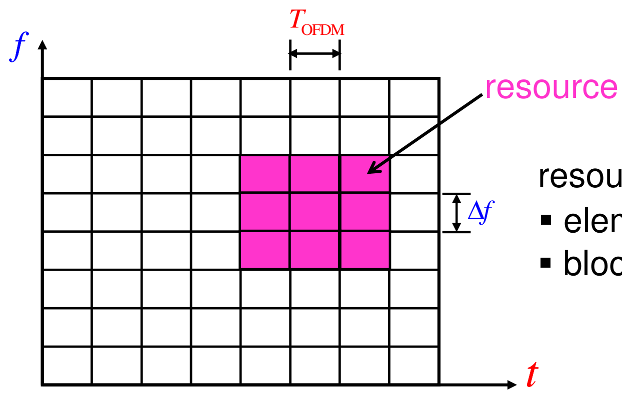
- OFDMA → orthogonal subcarriers
- TDMA → orthogonal time slots
- JT and JD (SDMA) →
  sharing of resources by $K \leq K_\mathrm{A}$ MTs allowed!



resource allocation
- element-wise
- block-wise

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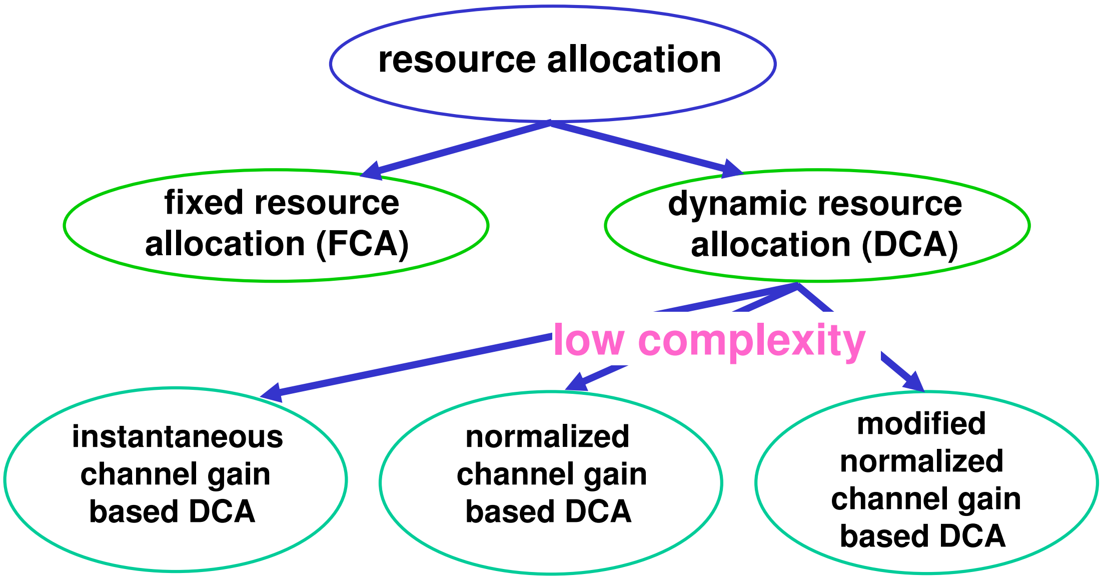
resource allocation

HS



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instantaneous channel gain based DCA

HS

high channel gains & wide spatial separations → **low BER**

$$\underline{\mathbf{G}}_{\text{inst}} = \begin{pmatrix} \left|\underline{H}^{(1,1)}\right|^2 & \cdots & \left|\underline{H}^{(1,K_\text{M})}\right|^2 \\ \vdots & & \vdots \\ \left|\underline{H}^{(K_\text{A},1)}\right|^2 & \cdots & \left|\underline{H}^{(K_\text{A},K_\text{M})}\right|^2 \end{pmatrix}$$

good BER performance
but unfair resource allocation

$$\begin{array}{c} \\ \text{AP1} \\ \text{AP2} \\ \text{AP3} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{MT1} & \text{MT2} & \text{MT3} & \text{MT4} & \text{MT5} & \text{MT6} \end{array} \\ \begin{pmatrix} 47 & 13 & 1 & 35 & 7 & 2 \\ 30 & 1 & 9 & 12 & 3 & 19 \\ 1 & 22 & 2 & 8 & 4 & 6 \end{pmatrix} \end{array}$$

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normalized channel gain based DCA

improve fairness

$$\underline{\mathbf{G}}_{\text{norm}} = \begin{pmatrix} \frac{\left|\underline{H}^{(1,1)}\right|^2}{\mathbf{E}\left\{\left|\underline{H}^{(1,1)}\right|^2\right\}} & \cdots & \frac{\left|\underline{H}^{(1,K_\mathrm{M})}\right|^2}{\mathbf{E}\left\{\left|\underline{H}^{(1,K_\mathrm{M})}\right|^2\right\}} \\ \vdots & & \vdots \\ \frac{\left|\underline{H}^{(K_\mathrm{A},1)}\right|^2}{\mathbf{E}\left\{\left|\underline{H}^{(K_\mathrm{A},1)}\right|^2\right\}} & \cdots & \boxed{\frac{\left|\underline{H}^{(K_\mathrm{A},K_\mathrm{M})}\right|^2}{\mathbf{E}\left\{\left|\underline{H}^{(K_\mathrm{A},K_\mathrm{M})}\right|^2\right\}}} \end{pmatrix}$$

fairness improvement
but BER performance degradation

only depends on fast fading

lose spatial information of MT positions

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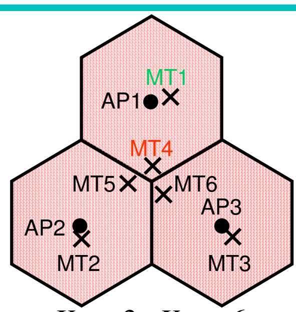
modified normalized channel gain based DCA

improve BER performance

$$\underline{\mathbf{G}}_{\text{m-norm}} = \begin{pmatrix} \frac{\left|\underline{H}^{(1,1)}\right|^2}{\sum_{k_\mathrm{A}=1}^{K_\mathrm{A}} \mathbf{E}\left\{\left|\underline{H}^{(k_\mathrm{A},1)}\right|^2\right\}} & \cdots & \frac{\left|\underline{H}^{(1,K_\mathrm{M})}\right|^2}{\sum_{k_\mathrm{A}=1}^{K_\mathrm{A}} \mathbf{E}\left\{\left|\underline{H}^{(k_\mathrm{A},K_\mathrm{M})}\right|^2\right\}} \\ \vdots & & \vdots \\ \frac{\left|\underline{H}^{(K_\mathrm{A},1)}\right|^2}{\sum_{k_\mathrm{A}=1}^{K_\mathrm{A}} \mathbf{E}\left\{\left|\underline{H}^{(k_\mathrm{A},1)}\right|^2\right\}} & \cdots & \frac{\left|\underline{H}^{(K_\mathrm{A},K_\mathrm{M})}\right|^2}{\sum_{k_\mathrm{A}=1}^{K_\mathrm{A}} \mathbf{E}\left\{\left|\underline{H}^{(k_\mathrm{A},K_\mathrm{M})}\right|^2\right\}} \end{pmatrix}$$



$K_\mathrm{A} = 3,\ K_\mathrm{M} = 6$

MT1:

$$\mathbf{E}\left\{\left|\underline{H}^{(1,1)}\right|^2\right\} >> \mathbf{E}\left\{\left|\underline{H}^{(2,1)}\right|^2\right\} \approx \mathbf{E}\left\{\left|\underline{H}^{(3,1)}\right|^2\right\}$$

$$\frac{\left|\underline{H}^{(1,1)}\right|^2}{\sum_{k_\mathrm{A}=1}^{3} \mathbf{E}\left\{\left|\underline{H}^{(k_\mathrm{A},1)}\right|^2\right\}} \approx \frac{\left|\underline{H}^{(1,1)}\right|^2}{\mathbf{E}\left\{\left|\underline{H}^{(1,1)}\right|^2\right\}}$$

MT4:

$$\mathbf{E}\left\{\left|\underline{H}^{(1,4)}\right|^2\right\} \approx \mathbf{E}\left\{\left|\underline{H}^{(2,4)}\right|^2\right\} \approx \mathbf{E}\left\{\left|\underline{H}^{(3,4)}\right|^2\right\}$$

$$\frac{\left|\underline{H}^{(1,4)}\right|^2}{\sum_{k_\mathrm{A}=1}^{3} \mathbf{E}\left\{\left|\underline{H}^{(k_\mathrm{A},4)}\right|^2\right\}} \approx \frac{\left|\underline{H}^{(1,4)}\right|^2}{3\,\mathbf{E}\left\{\left|\underline{H}^{(1,4)}\right|^2\right\}}$$

MT1 is more likely to be served by AP1 than MT4

main assumptions

- sub-area radius 250m
- dual slope average path gain (break even distance 300m, α_1=2.0, α_2=4.0)
- log-normal slow fading ($\sigma_G^2 = 16\mathrm{dB}$)
- Rayleigh fast fading
- perfect channel knowledge
- single resource
- uncoded transmission
- fixed modulation scheme: QPSK
- constant transmit power E_d
- noise considered $10\log\left(E_\mathrm{d}/\sigma^2\right) = 116\mathrm{dB}$
- P_b: instantaneous BER (averaging over noise & data, fixed channel)

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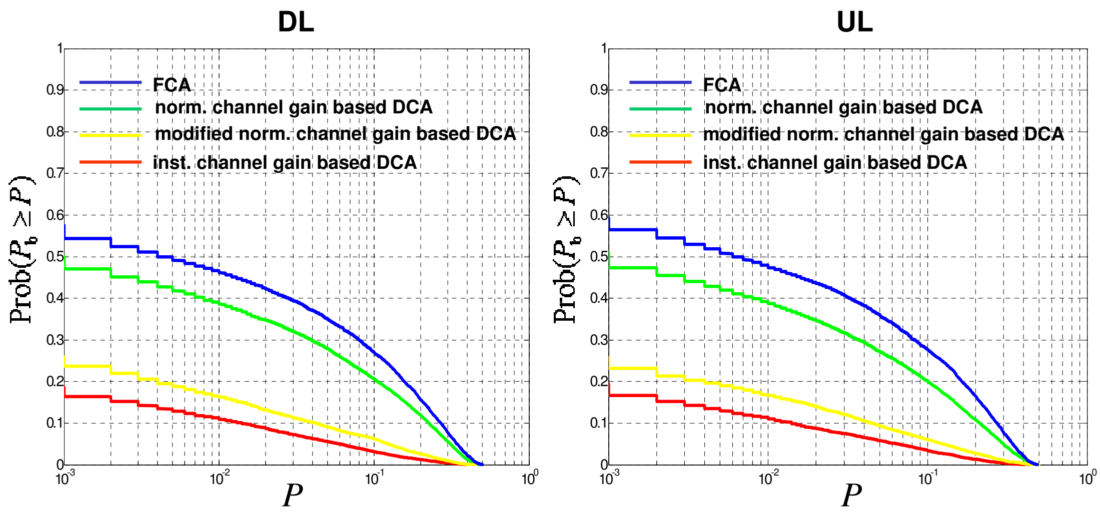
BER CCDFs

$$K_\mathrm{A} = K = 3,\ K_\mathrm{M} = 6,\ r = 3$$



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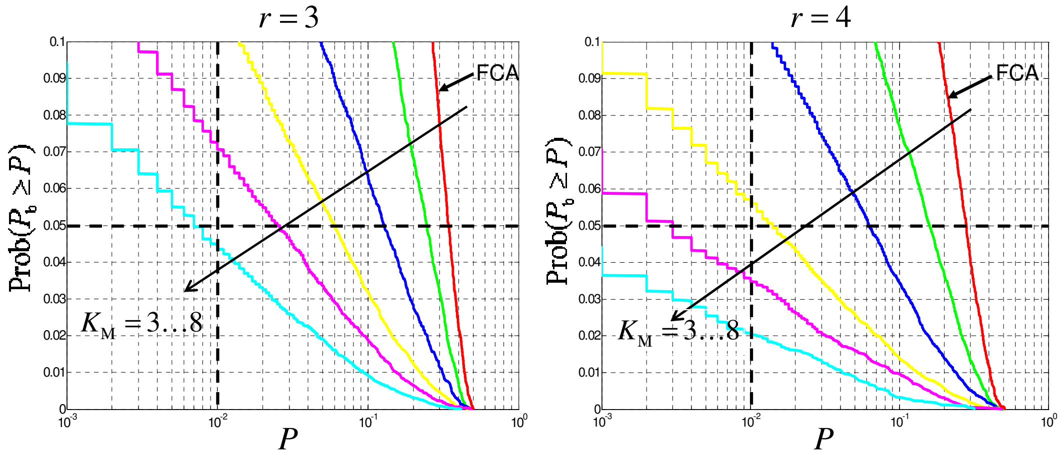
DL BER CCDFs of instantaneous channel gain based DCA

HS

$$K_A = K = 3,\ K_M = 3 \ldots 8$$



QoS requirement: $P_{out} = \mathrm{Prob}(P_b \geq P_{max} = 10^{-2}) \leq 0.05$

bigger K_M
bigger r
→ **better BER performance**

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spectrum efficiency of instantaneous channel gain based DCA

HS

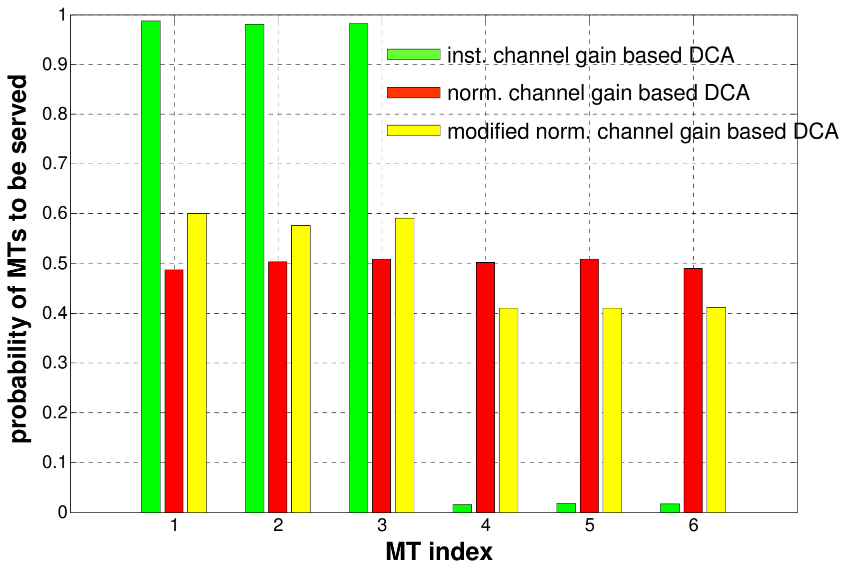
$$\eta / \mathrm{bps/Hz} = \frac{\frac{K}{K_A} \cdot R}{\Delta f \cdot r} \qquad \Delta f = 39.063\,\mathrm{kHz} \qquad R = 62.5\,\mathrm{kbps} \qquad K = K_A = 3$$

QoS requirement: $P_{out} = \mathrm{Prob}(P_b \geq P_{max} = 10^{-2}) \leq 0.05$

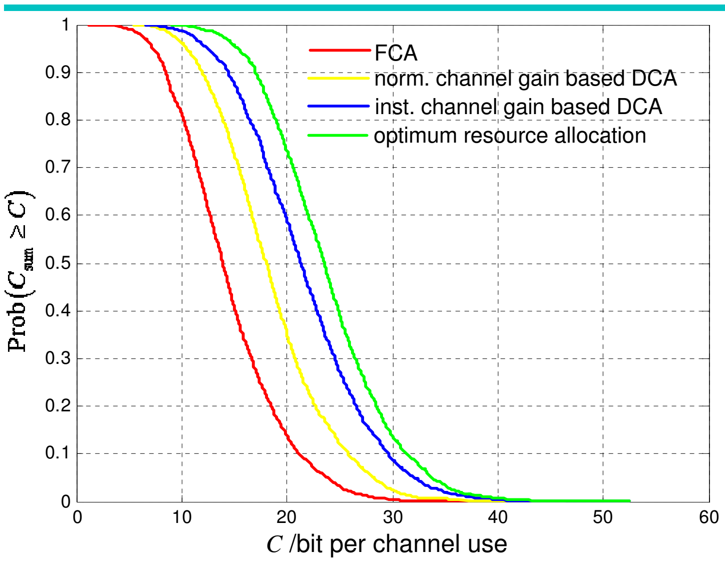
	$r=1$	$r=3$	$r=4$	$r=7$
$K_M = 3$	-	-	-	-
$K_M = 4$	-	-	-	-
$K_M = 5$	-	-	-	0.2286
$K_M = 6$	-	-	-	0.2286
$K_M = 7$	-	-	0.4000	0.2286
$K_M = 8$	-	0.5333	0.4000	0.2286

enough MTs
↓
max. η : 1.6 bps/Hz ($r = 1$)

fairness comparison



UL sum capacity CCDFs



$$K_{\mathrm{A}} = K = 1$$

$$K_{\mathrm{M}} = 2$$

$$r = 1$$

$$K_{\mathrm{S}} = 9$$

$$\left(\frac{K_{\mathrm{M}}!}{K!(K_{\mathrm{M}} - K)!}\right)^{K_{\mathrm{S}}} = 2^9 = 512$$

optimum resoure allocation: $\max\{C_{\mathrm{sum}}\} = \max\left\{\sum_{i=1}^{K_{\mathrm{S}}} \sum_{k=1}^{K} C_k^{(i)}\right\}$

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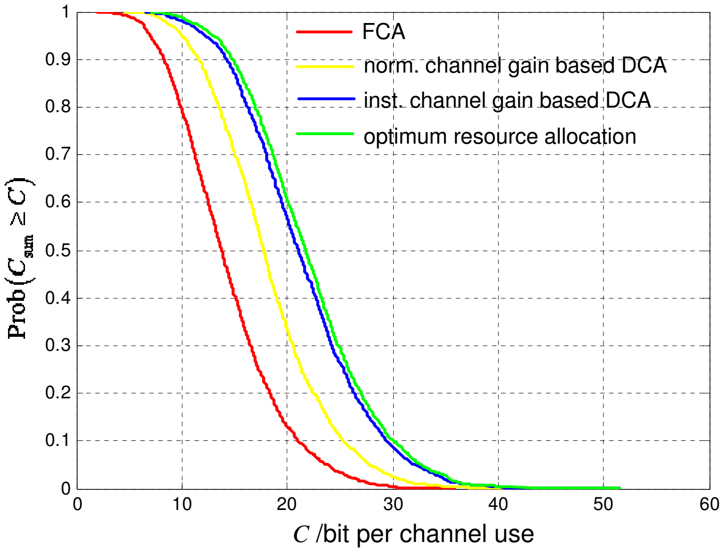
DL sum capacity CCDFs

HS



$K_{\mathrm{A}} = K = 1$

$K_{\mathrm{M}} = 2$

$r = 1$

$K_{\mathrm{S}} = 9$

instantaneous channel gain based DCA is near-optimum!

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summary

HS

	inst. channel gain based DCA	norm. channel gain based DCA	modified norm. channel gain based DCA
BER/ capacity	very good	bad	good
fairness	bad	very good	good

Receiver oriented FEC coding (RFC) for selective channels

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Email: jhahn@rhrk.uni-kl.de

goal

minimize word error probability

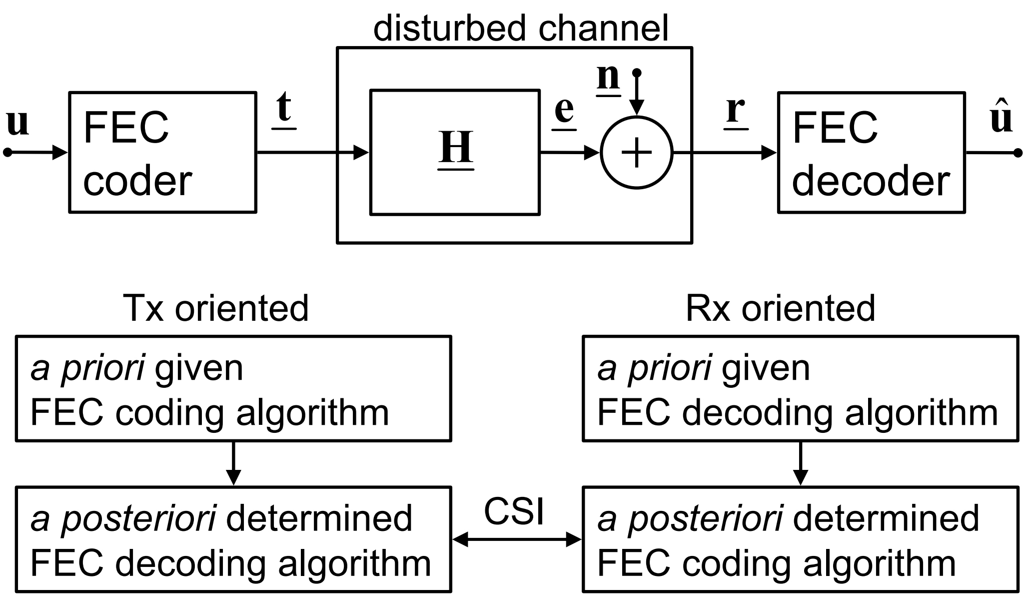
- for given Tx energy
- for fixed decoding algorithm
  (Rx oriented FEC coding (RFC))
- under exploitation of CSI at Tx side

schemes for Rx orientation

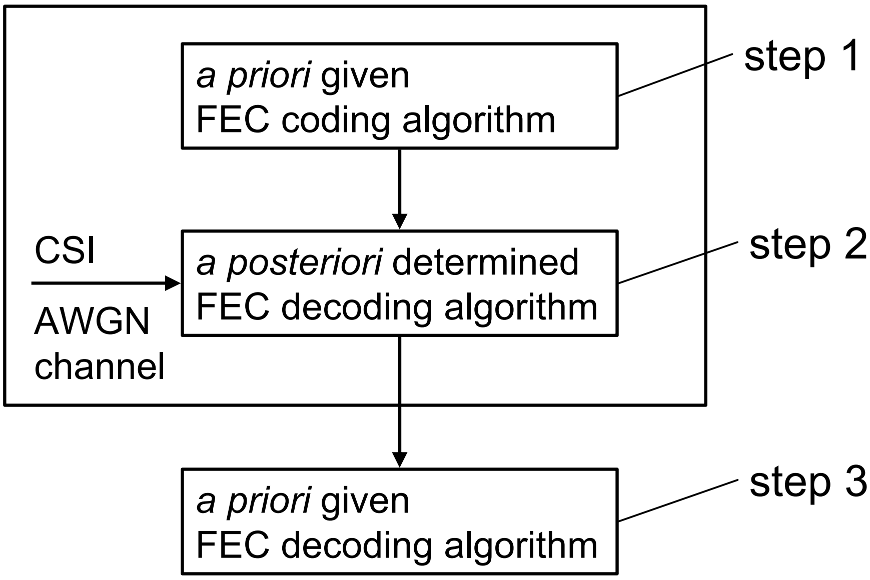
- Tx signal power adaptation wrt. channel
  Receiver Oriented FEC Coding

- Precoding, i.e. Costa/TH
  Transmit Non-linear Zero Forcing (TxNZF)

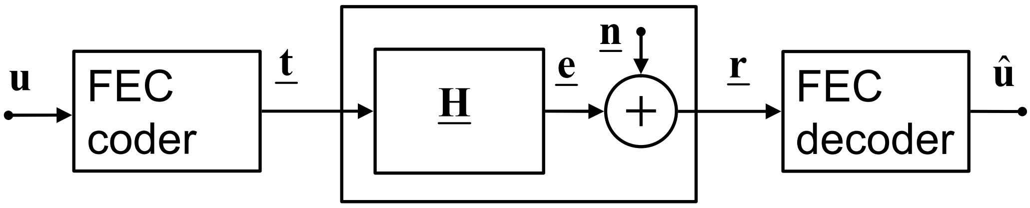
coded transmission schemes



a priori determination of FEC decoder



selective channel



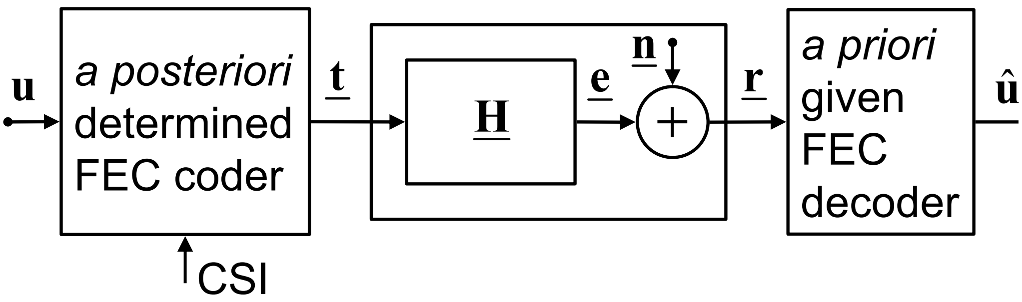
uncoded data: $\mathbf{u} = \left(u_1 \ldots u_N\right)^{\mathrm{T}} \in \{-1,1\}^{N\times 1}$

channel transfer function: $\underline{\mathbf{h}} = \left(\underline{h}_1 \ldots \underline{h}_Q\right)^{\mathrm{T}} \in \mathbb{C}^{Q\times 1}$

e.g.: OFDM or time-variant single carrier

coded data: $\underline{\mathbf{t}} = \left(\underline{t}_1 \ldots \underline{t}_Q\right) \in \begin{cases} \mathbb{V}_{\mathrm{D}}^{Q\times 1} & \rightarrow \text{conventional} \\ \mathbb{C}^{Q\times 1} & \rightarrow \text{Rx oriented} \end{cases}$

Rx oriented FEC coding (RFC)



uncoded data vectors: $\mathbf{u} \in \{-1;1\}^{N\times 1}$, $\hat{\mathbf{u}} \in \{-1;1\}^{N\times 1}$

coded Tx signal: $\underline{\mathbf{t}} \in \mathbb{C}^{Q\times 1}$

Tx energy: $T = \frac{1}{2}\underline{\mathbf{t}}^{\mathrm{H}}\underline{\mathbf{t}} = \text{const.}$

power adaptation methods

- Powerloading: adaptation irrespective of error correction capability of FEC code

minimize: $\sum_{q=1}^{Q} P_q$

- RFC: consideration of error correction capability of FEC code

minimize: $P_{\mathrm{w}} =$

$$1-\left[\prod_{q=1}^{Q}\left(1-P_q\right)+\sum_{\upsilon=1}^{t}\left\{\sum_{\mu_1=1}^{Q}P_{\mu_1}\sum_{\mu_2=\mu_1+1}^{Q}P_{\mu_2}\cdots\sum_{\mu_\upsilon=\mu_{\upsilon-1}+1}^{Q}P_{\mu_\upsilon}\prod_{\substack{q=1\\ q\neq\{\mu_1\ldots\mu_\upsilon\}}}^{Q}\left(1-P_q\right)\right\}\right]$$

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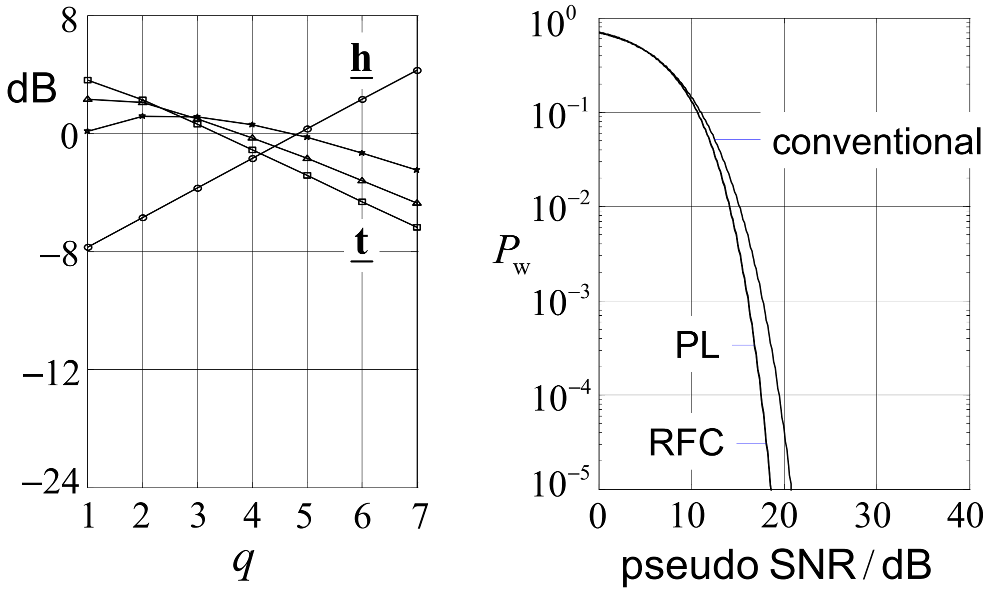
example (7,4) Hamming code



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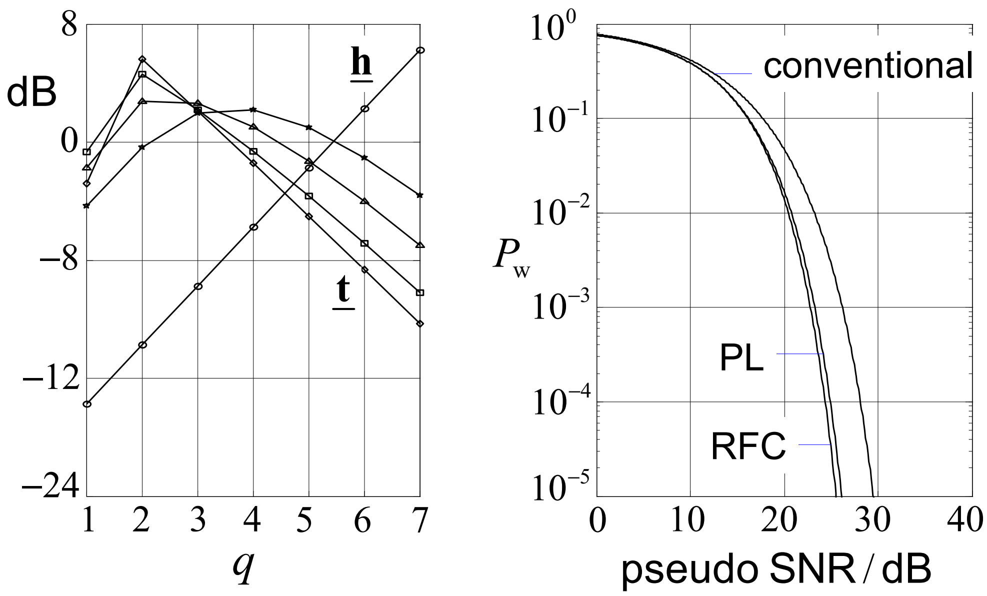
example (7,4) Hamming code



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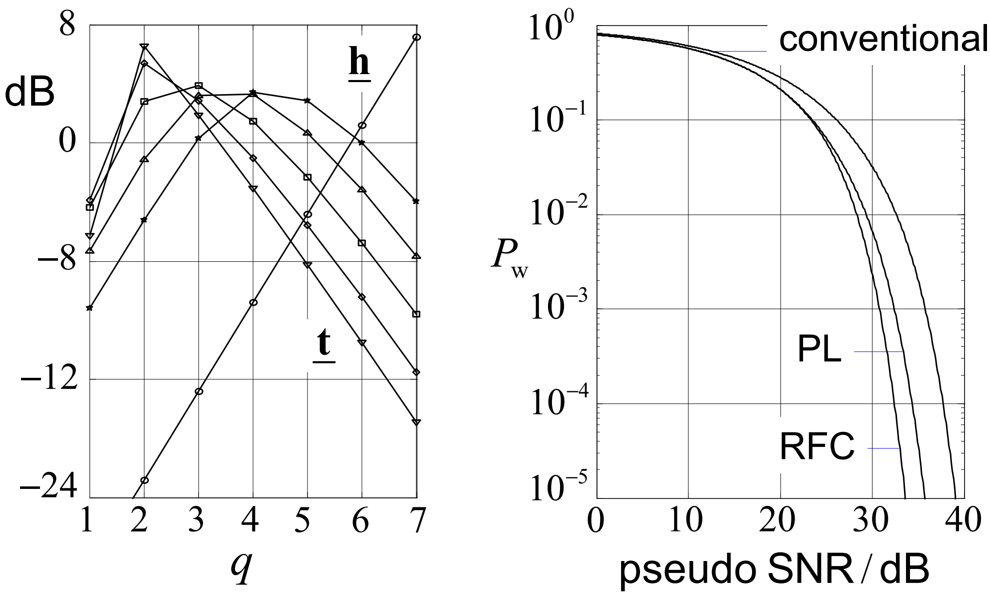
example (7,4) Hamming code



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summary

exploitation of FEC code capability:

- neglect/give up components with the largest attenuations (cf. Water-filling)
- compensate attenuation of remaining components (cf. Powerloading or Zero-Forcing)
- number of neglected/given up components depends on number of correctable errors

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

Clemens Stierstorfer **Robert Fischer**

LEHRSTUHL FÜR INFORMATIONSÜBERTRAGUNG
Laboratorium für Nachrichtentechnik — Professor Dr.–Ing. J. Huber
Friedrich–Alexander–Universität Erlangen–Nürnberg

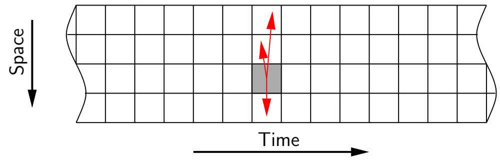
6. Sitzung ITG Fachgruppe "Angewandte Informationstheorie", 7. Oktober 2005, Kaiserslautern

Overview

- Introduction — MIMO ISI channel
- Uncoded transmission
  - Multicarrier (MC) transmission
  - Singlecarrier (SC) transmission
- Coded transmission
  - Implementation
  - Numerical results
  - Observations and interpretation
- Summary and outlook

Introduction — MIMO ISI Channel

2



Channel Model:

- point-to-point transmission with K transmit/receive antennas

  $\Rightarrow$ *multiantenna interference (MAI)*

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

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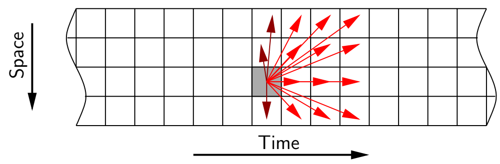
Introduction — MIMO ISI Channel

2



Channel Model:

- point-to-point transmission with K transmit/receive antennas

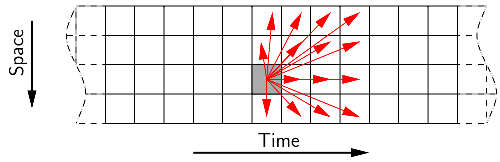
  $\Rightarrow$ *multiantenna interference (MAI)*

- dispersive channels

  $\Rightarrow$ *intersymbol interference (ISI)*

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

Introduction — MIMO ISI Channel



Channel Model:

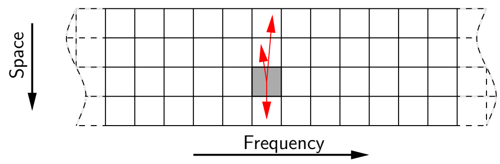
- point-to-point transmission with K transmit/receive antennas
  ⇒ *multiantenna interference (MAI)*
- dispersive channels
  ⇒ *intersymbol interference (ISI)*
  ⇒ *multiple-input/multiple-output ISI channel*

Task: ⇒ *cancellation/equalization of interference!*

Assumptions:

- block fading
- no interblock interference
- additive, white Gaussian noise

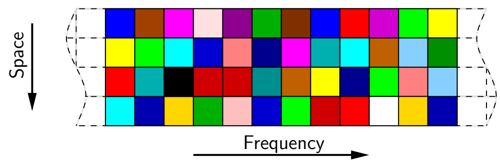
Uncoded Transmission — Multicarrier



MIMO OFDM:

- decomposition of dispersive channel into D non-dipersive MIMO sub-channels (carriers)
  ⇒ *separation of ISI/MAI cancellation/equalization*

Uncoded Transmission — Multicarrier

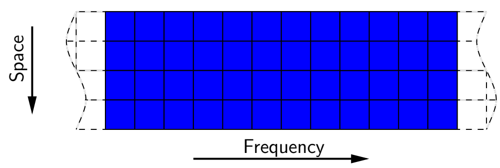


MIMO OFDM:

- decomposition of dispersive channel into D non-dipersive MIMO sub-channels (carriers)
  - ⇒ *separation of ISI/MAI cancellation/equalization*
- MIMO detection in each carrier (e.g. indiv. sort. DFE (V-BLAST))
  - ⇒ *decomposition into DK independent sub-channels*

Uncoded Transmission — Multicarrier

3



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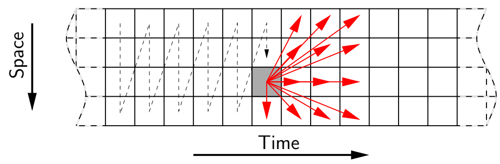
  - ⇒ *decomposition into DK independent sub-channels*

Loading:

- adapt rate and power distribution according to SNRs
- perfect loading:
  - ⇒ *max. DK channels with equal BER*

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

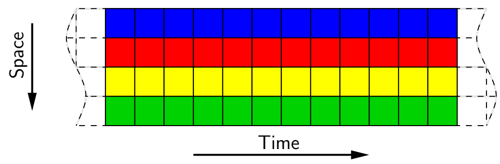
Uncoded Transmission — Singlecarrier



Spatio-temporal DFE (MIMO DFE):

- transformation of channel in spatio-temporal causal form
  - $\Rightarrow$ *joint cancellation/equalization of ISI and MAI*

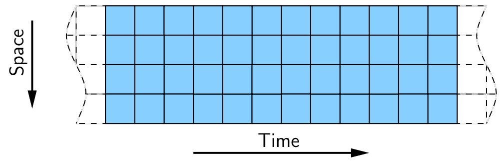
Uncoded Transmission — Singlecarrier



Spatio-temporal DFE (MIMO DFE):

- transformation of channel in spatio-temporal causal form
  - $\Rightarrow$ *joint cancellation/equalization of ISI and MAI*
  - $\Rightarrow$ *decomposition into K independent sub-channels*

Uncoded Transmission — Singlecarrier



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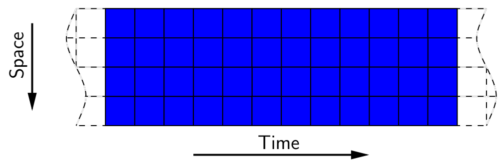
Uncoded Transmission — Singlecarrier

4



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Loading:

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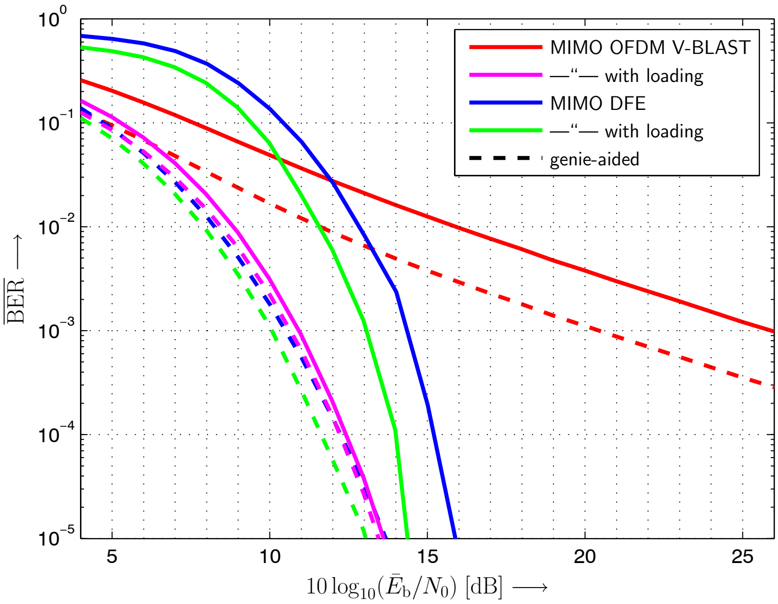
- perfect loading:
  - ⇒ *max. K channels with equal* BER

Comparison with MIMO OFDM:

⇒ *for ($D \to \infty$ & genie-aided) equal performance!*

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

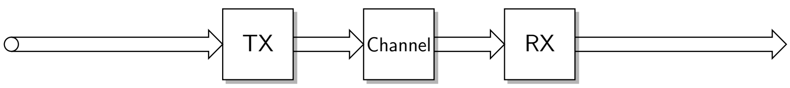
Uncoded Transmission — Numerical Results



Parameters:

- $K = 4$ antennas
- $L = 4$ length of channel impulse response
- equal gain PDP
- $D = 256$ carriers
- 16-QAM

Coded Transmission — Motivation

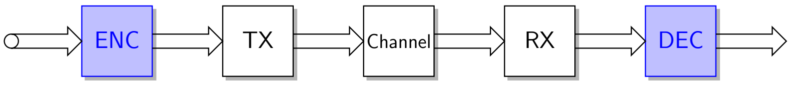


So far:

- uncoded transmission

Coded Transmission — Motivation

6



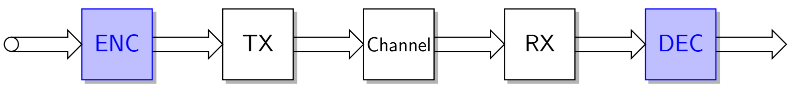
So far: ■ uncoded transmission

Now: → *introduction of channel coding*

Constraints:
- coding restricted to a single (OFDM) block
- fixed data rate
- simple coding schemes (here: BICM), non-iterative

Coded Transmission — Motivation

6



So far: ■ uncoded transmission

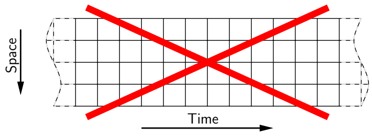
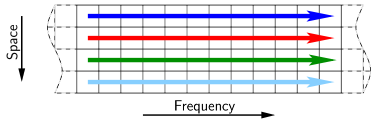
Now: → *introduction of channel coding*

Constraints:
- coding restricted to a single (OFDM) block
- fixed data rate
- simple coding schemes (here: BICM), non-iterative

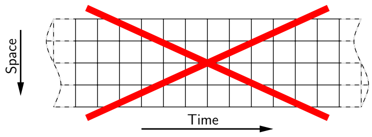
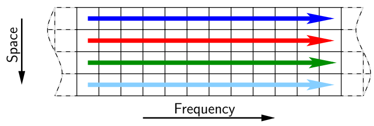
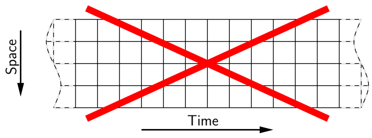
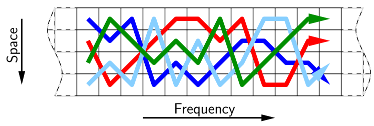
Open Questions:
- arrangement of codeword(s) within a block
- required channel state information at TX/RX

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

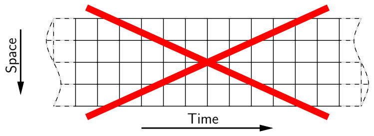
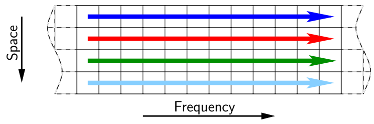
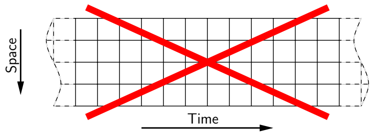
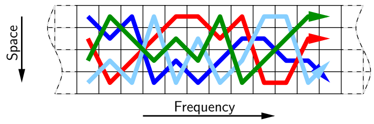
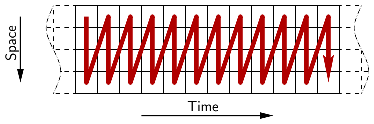
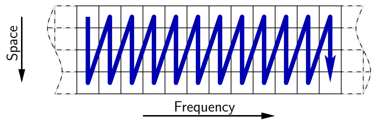
Coded Transmission — Implementation

TX CSI	Singlecarrier	Multicarrier
No	 Coding + spatio-temporal DFE not applicable	 DFE with/no sorting of codewords

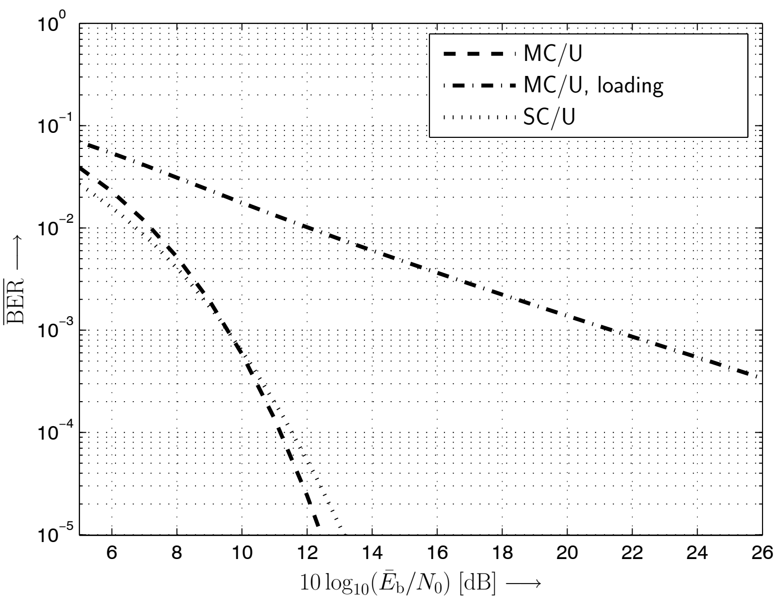
Coded Transmission — Implementation

TX CSI	Singlecarrier	Multicarrier
No	 Coding + spatio-temporal DFE not applicable	 DFE with/no sorting of codewords
Partial	 Coding + spatio-temporal DFE not applicable	 V-BLAST in carriers with/no loading

Coded Transmission — Implementation

TX CSI	Singlecarrier	Multicarrier
No	 Coding + spatio-temporal DFE not applicable	 DFE with/no sorting of codewords
Partial	 Coding + spatio-temporal DFE not applicable	 V-BLAST in carriers with/no loading
Full	 Spatio-temporal precoding (with/no loading)	 SVD in carriers (with/no loading) Precoding in carriers (with/no loading)

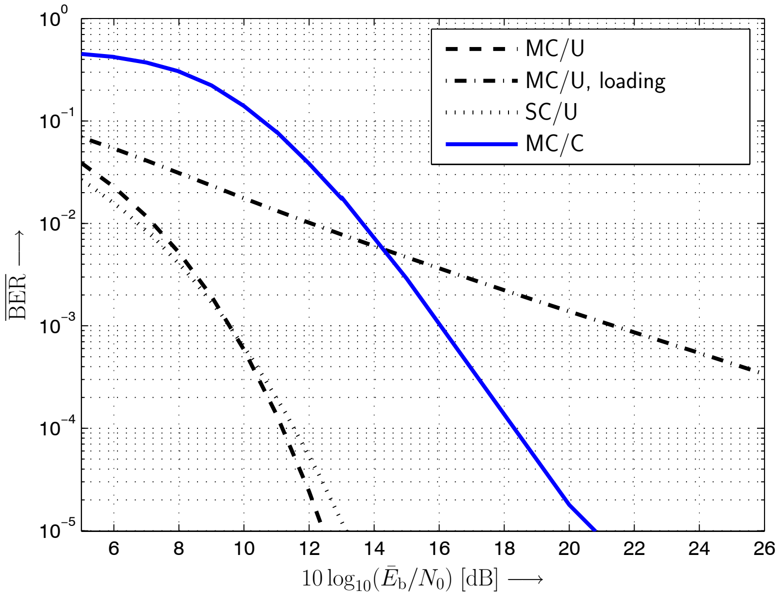
Coded Tranmission — Numerical Results I



Parameters:

- $K = 4$ antennas
- $L = 4$ length of channel impulse response
- equal gain PDP
- $D = 256$ carriers
- 16-QAM uncoded
- 256-QAM coded
- convolutional code:
  - rate $1/2$
  - 64 states

Coded Tranmission — Numerical Results I



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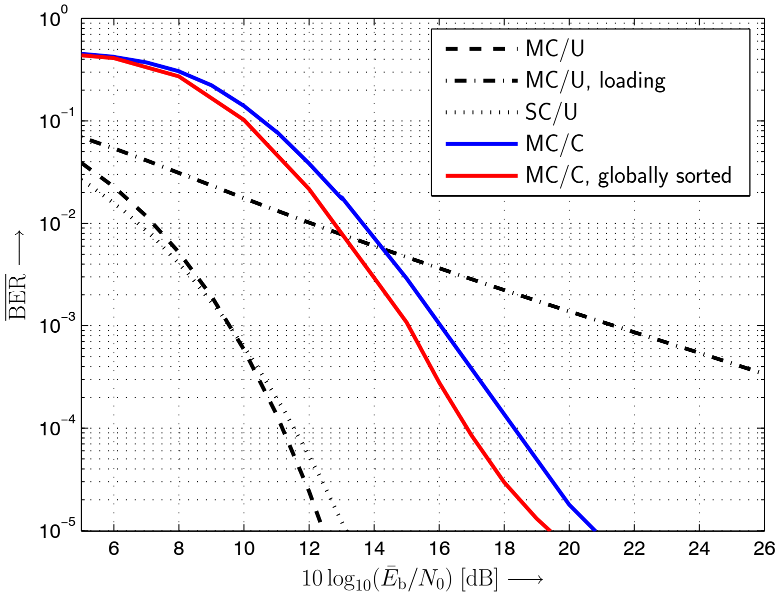

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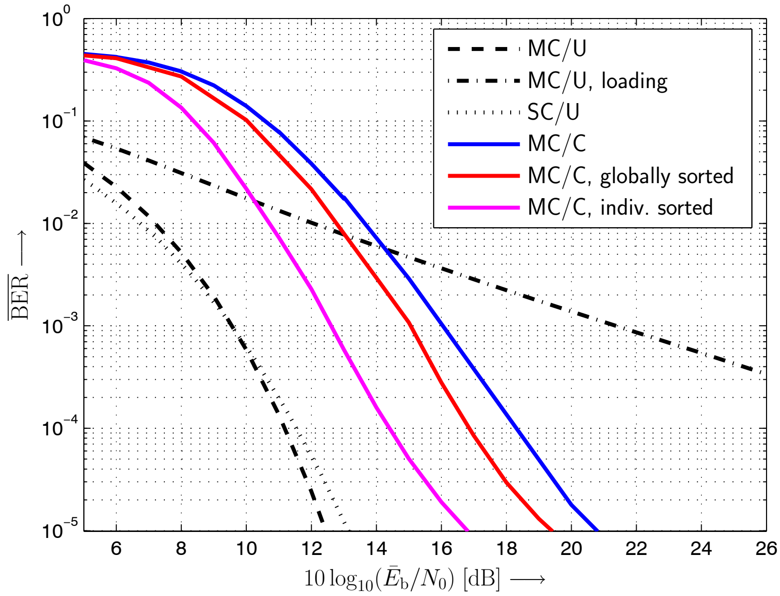

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Coded Tranmission — Numerical Results I

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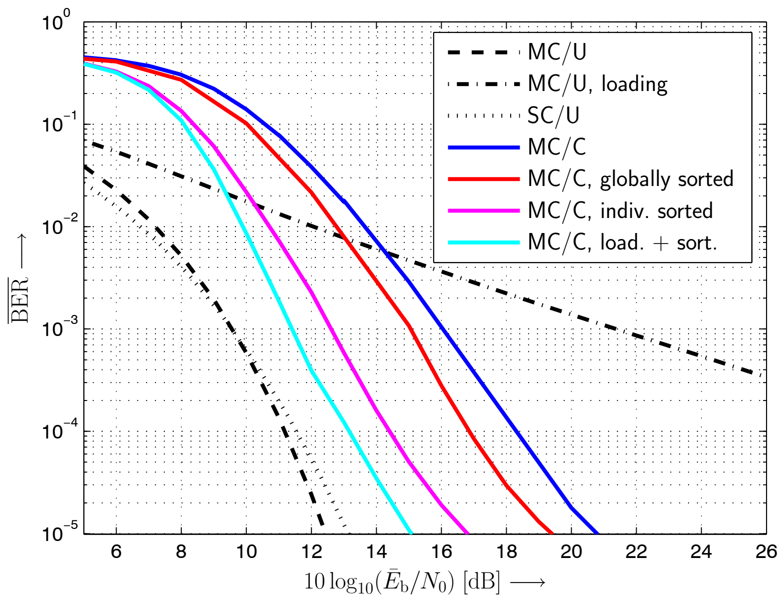

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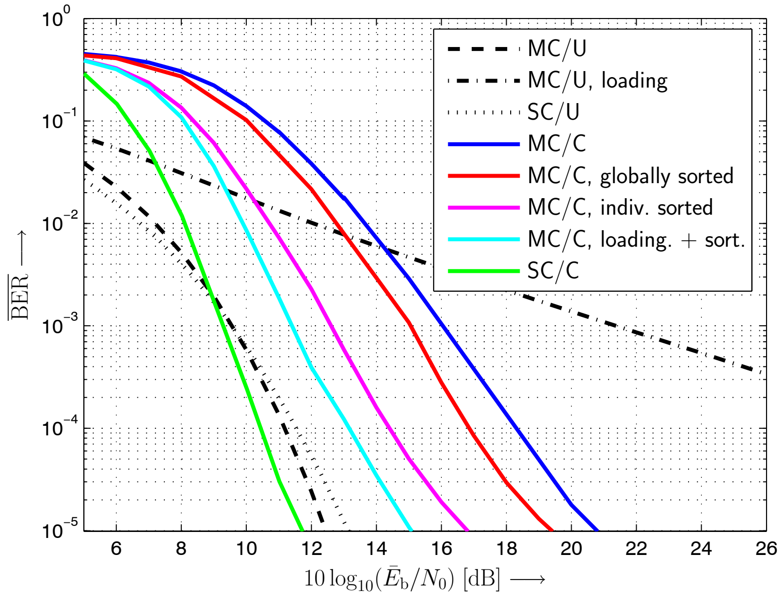
Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

Coded Tranmission — Numerical Results I



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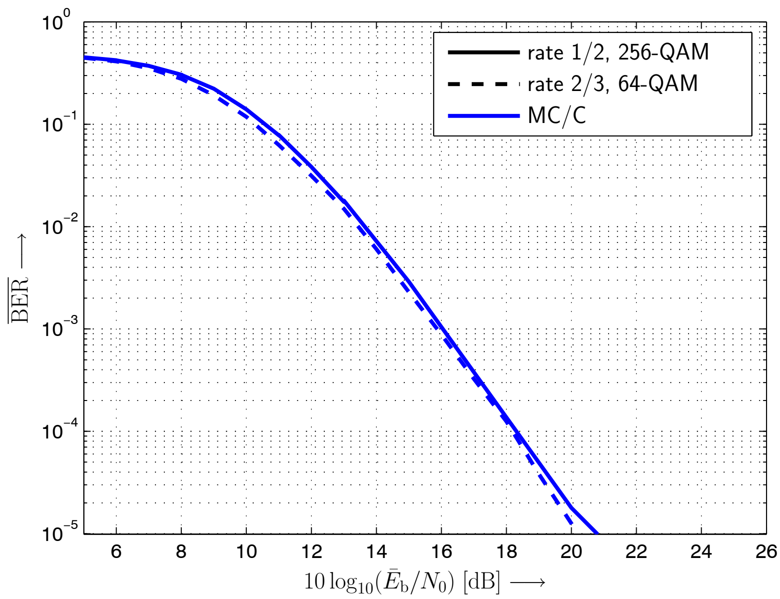
Coded Tranmission — Numerical Results II



Parameters:

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- $D = 256$ carriers
- 16-QAM uncoded
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- convolutional code:
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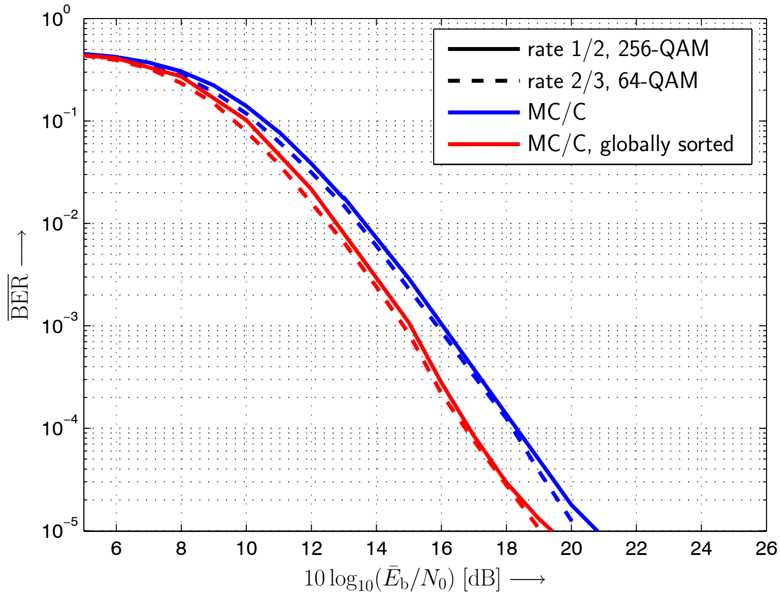
  - 64 states

Coded Tranmission — Numerical Results II



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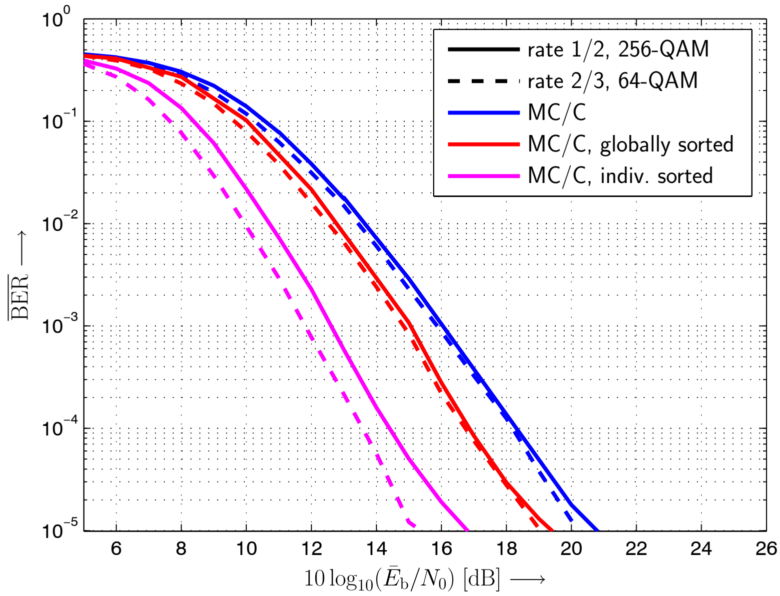

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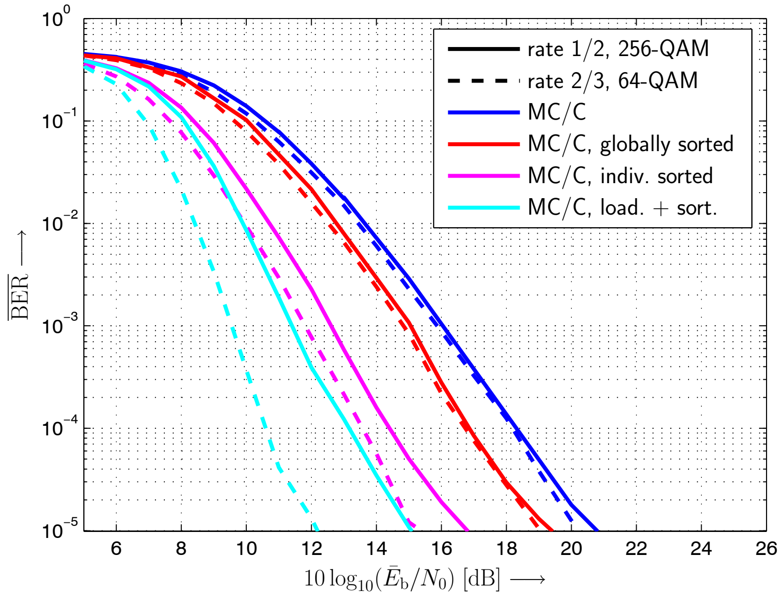

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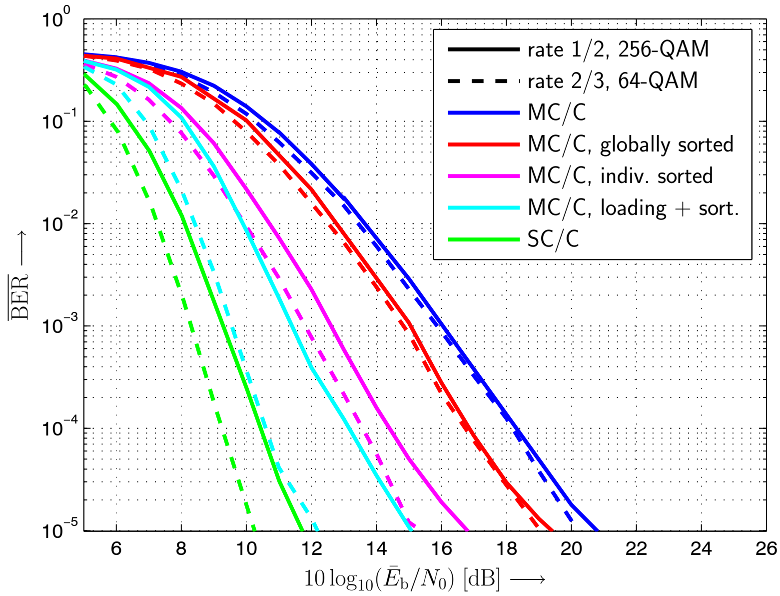

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Coded Transmission — Observations

Observations:

⇒ *sorting is essential*

⇒ *TX CSI improves performance*

⇒ *BICM cannot take significant advantages of loading*

⇒ *BICM with rate-$1/2$ worse than with rate-$2/3$*

Coded Transmission — Observations

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Observations:

⇒ *sorting is essential*

⇒ *TX CSI improves performance*

⇒ *BICM cannot take significant advantages of loading*

⇒ *BICM with rate-$1/2$ worse than with rate-$2/3$*

But: ⇒ *coded transmission hardly beats uncoded transmission with TX CSI!*

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

MIMO OFDM:

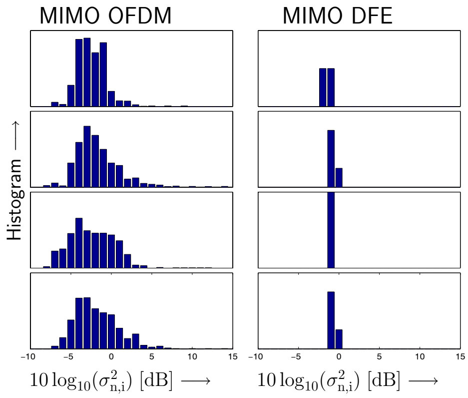
- e.g. V-BLAST in carriers
- $256 \cdot 4$ sub-channels
- different SNRs in sub-channels

⇒ virtual “fading” scenario

MIMO DFE:

- 4 sub-channels

⇒ only moderate “fading”



Coded Transmission — Interpretation I 11

MIMO OFDM:

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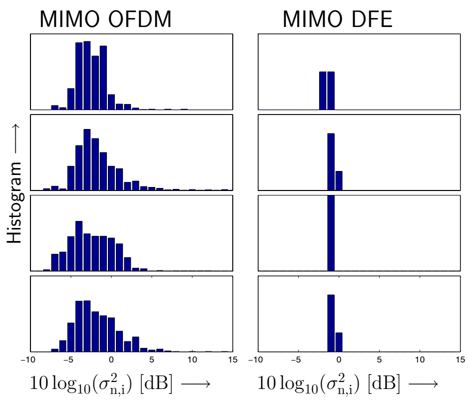


Effect: *⇒ coding works better with MIMO DFE!*

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

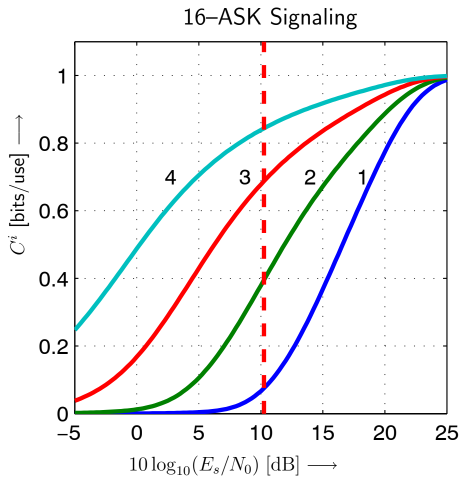
Coded Transmission — Interpretation II

Parameters:

- 16-QAM (uncoded)
- rate $1/2$, 256-QAM (coded)

⇒ 4 bits/symbol/dimension

$$c_1, c_2, c_3, c_4 \rightarrow \boxed{\mathcal{M}} \rightarrow a$$



Considering Capacities:

- coding levels with high variance of sub-capacities

Coded Transmission — Interpretation II

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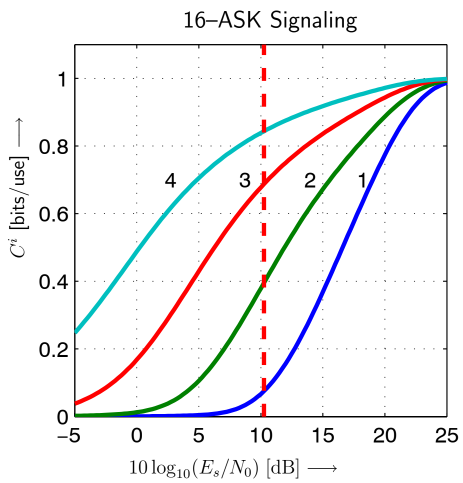
Parameters:

- 16-QAM (uncoded)
- rate $1/2$, 256-QAM (coded)

⇒ 4 bits/symbol/dimension

$$c_1, c_2, c_3, c_4 \rightarrow \boxed{\mathcal{M}} \rightarrow a$$



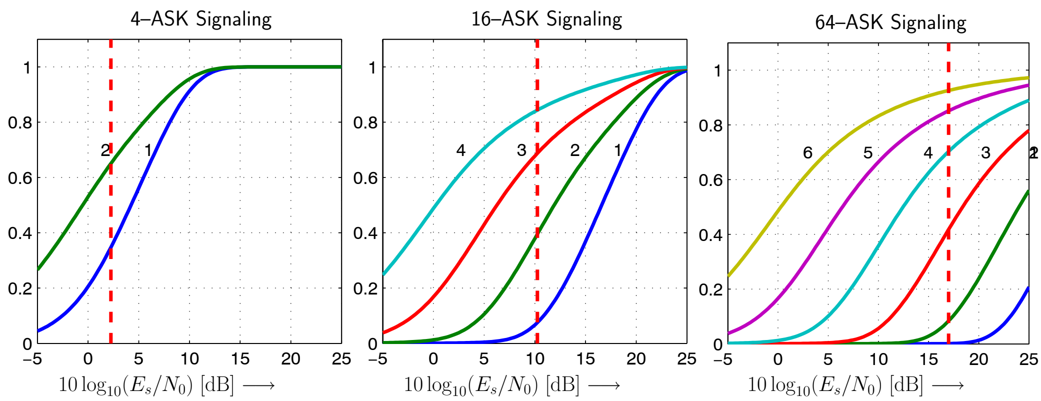
Considering Capacities:

- coding levels with high variance of sub-capacities

Effect: *⇒ small sub-capacities decrease performance!*

Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency–Selective MIMO Channels

Coded Transmission — Interpretation III



⇒ *variance of sub-capacities increases significantly!*

⇒ *loading aggravates effect!*

Thus: ⇒ *performance gains due to loading are limited!*

Summary and Outlook

Comparison of Code Design Requirements:

⇒ *TX CSI is vital*

⇒ *singlecarrier outperforms multicarrier*

⇒ *BICM combined with large signal constellations not recommendable*

Summary and Outlook

Comparison of Code Design Requirements:

=> *TX CSI is vital*

=> *singlecarrier outperforms multicarrier*

=> *BICM combined with large signal constellations not recommendable*

Topic of further Research:

- interaction of channel coding and loading
- adapt metric calculation
- suited low-complexity coding schemes
- spreading over carriers (MC-CDMA)

ITG-Fachgruppe „Angewandte Informationstheorie“,
Kaiserlautern, 07. Oktober 2005

Unbiased Semiblind Channel Estimation for Coded Systems

Dipl.-Ing. Ansgar Scherb,
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Universität Bremen
Arbeitbereich Nachrichtentechnik (ANT)

Universität Bremen

Arbeitsbereich Nachrichtentechnik

Universität Bremen

Overview

1. Motivation
2. System Model
3. Semi-Blind Channel Estimation (SBCE) Methods
4. Numerical Results
5. Conclusion

Universität Bremen

1. Motivation

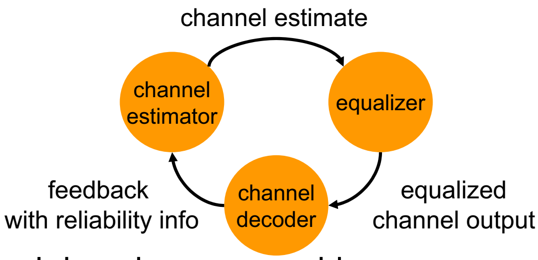
$N_{Tx} \times N_{Rx}$	Chan. len. (L+1)	Number of coefficients to be estimated	minimum number of pilot symbols
1x1	1	1	~ 1
1x1	5	5	~ 5
4x4	5	80	~ 20
8x8	5	320	~ 40

- **[Marzetta 99]:** „ (…), when trying to achieve the maximum possible throughput, half of the possible intervall is used for training “
- **[Hassibi, Hochwald 2000]:** „(…) training based schemes are highly suboptimal at low SNR“

→ **Drastical reduction of the number of training symbols is required (blind or semi-blind)**

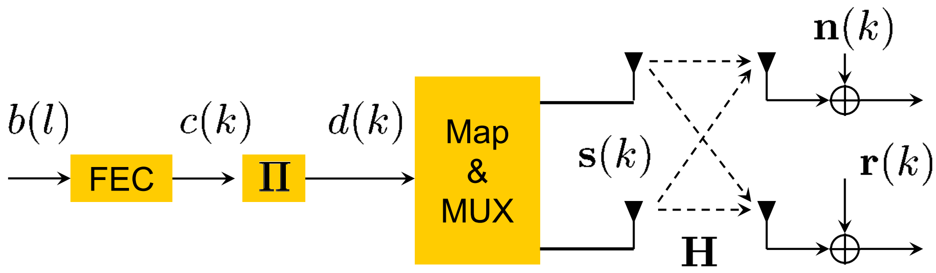
1. Motivation

- *Target:* short training ↔ accurate estimates
- Therefore, iterative channel estimation (semi-blind)



- Channel decoder may provide
  - different qualities of feedback data (intrinsic, extrinsic and a-posteriori)
  - reliability information

2.1 System Model: Transmitter & Channel



- $N_\mathrm{R} \times N_\mathrm{T}$ MIMO channel stays constant within a block of K symbol periods
- Each data block $\mathbf{S} = [\mathbf{S}_\mathrm{P},\ \mathbf{S}_\mathrm{I}]$ consists of K_I pilot vectors and K_P information vectors with $K = K_\mathrm{P} + K_\mathrm{I}$
- System model

$$\boxed{\begin{gathered}\mathbf{R} = \mathbf{H}\mathbf{S} + \mathbf{N} \\ [\mathbf{R}_\mathrm{P},\ \mathbf{R}_\mathrm{I}] = \mathbf{H}[\mathbf{S}_\mathrm{P},\ \mathbf{S}_\mathrm{I}] + [\mathbf{N}_\mathrm{P},\ \mathbf{N}_\mathrm{I}]\end{gathered}}$$

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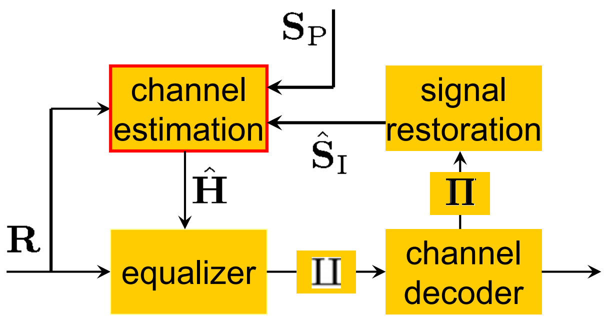
2.2 System Model: Receiver



- $\hat{\mathbf{S}}_\mathrm{I}$: hard decided feedback with $\hat{\mathbf{S}} = [\mathbf{S}_\mathrm{P},\ \hat{\mathbf{S}}_\mathrm{I}]$
- $\mathbf{E} = \mathbf{S} - \hat{\mathbf{S}}$: error with bit error probability (BEP) P_b
- Approx.:

$$\boxed{\begin{aligned}&\mathbf{S}\mathbf{S}^H \approx \hat{\mathbf{S}}\hat{\mathbf{S}}^H \approx K\mathbf{I}_{N_\mathrm{T}} \\ &\mathbf{S}\hat{\mathbf{S}}^H \approx \hat{\mathbf{S}}\mathbf{S}^H \approx (K_\mathrm{P} + (1 - 2 \cdot P_b)K_\mathrm{I})\mathbf{I}_{N_\mathrm{T}}\end{aligned}}$$

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2.3 Intrisnsic, Extrinsic, and A-Posteriori Feedback

- ***intrinsic*** **feedback:** $s_n(k)$ only depends on corresponding observation $\mathbf{r}(k)$
  - noise and feedback error are correlated

$$\boxed{\mathbf{C}_{NE} = \mathrm{E}\{\mathbf{N}\mathbf{E}_{\mathrm{intr.}}^H\} \neq \mathbf{0}}$$

- ***extrinsic*** **feedback:** $s_n(k)$ depends on non-corresponding observations $\mathbf{r}(\kappa)$; $\kappa \neq k$
  - noise and feedback error are uncorrelated

$$\boxed{\mathbf{C}_{NE} = \mathrm{E}\{\mathbf{N}\mathbf{E}_{\mathrm{extr.}}^H\} = \mathbf{0}}$$

- ***a-posteriori*** **feedback:** combination of intrinsic and extrinsic information
  - noise and feedback error are correlated
  - most reliable feedback

3.1 Unbiased Pilot Assisted CE (*E0*)

- Estimator *E0*: $\boxed{\hat{\mathbf{H}}_0 = \mathbf{R}\mathbf{S}^H(\mathbf{S}\mathbf{S}^H)^{-1}}$
- Unbiasedness: $\boxed{\mathrm{E}\{\hat{\mathbf{H}}_0\} = \mathbf{H}}$
- MSE: $\boxed{\mathrm{E}\{\|\mathbf{H} - \hat{\mathbf{H}}_0\|_F^2\} \approx \begin{cases} \sigma^2 \frac{N_\mathrm{R} \cdot N_\mathrm{T}}{K} \\ \sigma^2 \frac{N_\mathrm{R} \cdot N_\mathrm{T}}{K_\mathrm{P}} \end{cases}}$

- any symbol is a-priori known at the transmitter
- asymptotic CRLB for *high* SNR

- only pilots are used for CE
- asymptotic CRLB for *low* SNR

3.2 Biased SBCE (*E1*)

- Estimator *E1*: $\boxed{\hat{\mathbf{H}}_1 = \mathbf{R}\hat{\mathbf{S}}^H(\hat{\mathbf{S}}\hat{\mathbf{S}}^H)^{-1}}$
- Unbiasedness: $\boxed{\mathrm{E}\{\hat{\mathbf{H}}_1\} = \underbrace{\frac{K_\mathrm{P}+(1-2P_b)K_\mathrm{I}}{K}}_{\text{scaling factor}}\mathbf{H} + \underbrace{\mathbf{C}_{NE}}_{\text{=0 for extrinsic feedback}}}$
  - *E1* is biased for $P_b > 0$ and unbiased for $P_b \to 0$
- MSE for *E1* with extrinsic feedback:

$$\boxed{\mathrm{E}\{\|\mathbf{H} - \hat{\mathbf{H}}_1\|_F^2\} = \left(\frac{2P_bK_\mathrm{I}}{K}\right)^2 \mathbf{H} + \sigma^2 \frac{N_\mathrm{R}\cdot N_\mathrm{T}}{K}}$$

  - tends to $\sigma^2 \frac{N_\mathrm{R}\cdot N_\mathrm{T}}{K}$ for very high and low SNR

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3.3 Unbiased SBCE with Extrinsic Feedback (*E2*)

- Estimator *E2*:

$$\boxed{\hat{\mathbf{H}}_2 = \frac{K}{K_\mathrm{P}+(1-2P_b)K_\mathrm{I}}\mathbf{R}\hat{\mathbf{S}}^H(\hat{\mathbf{S}}\hat{\mathbf{S}}^H)^{-1}}$$

- MSE for *E1* with extrinsic feedback:

$$\boxed{\mathrm{E}\{\|\mathbf{H} - \hat{\mathbf{H}}_2\|_F^2\} = \sigma^2 \frac{K N_\mathrm{R}\cdot N_\mathrm{T}}{(K_\mathrm{P}+(1-2P_b)K_\mathrm{I})^2}}$$

  - tends to $\sigma^2 \frac{N_\mathrm{R}\cdot N_\mathrm{T}}{K}$ for $P_b \to 0$
  - may become large for $P_b \gg 0$

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3.4 Unbiased SBCE with Extrinsic Feedback (*E3*)

- Estimator *E3*:

$$\boxed{\hat{\mathbf{H}}_3(a) = K\left(\frac{a}{K_\mathrm{P}}\mathbf{R}_\mathrm{P}\mathbf{S}_\mathrm{P}^H + \frac{1-a}{(1-2P_b)K_\mathrm{I}}\mathbf{R}_\mathrm{I}\hat{\mathbf{S}}_\mathrm{I}^H\right)(\hat{\mathbf{S}}\hat{\mathbf{S}}^H)^{-1}}$$

  - $a \in [0,1]$: factor weighting pilot and feedback part
  - for any a *E3* is unbiased
  - optimum value in terms of minimum MSE:

$$\boxed{a_\mathrm{opt.} = \frac{K_\mathrm{P}}{K_\mathrm{P}+(1-2Pb)^2K_\mathrm{I}}}$$

- MSE:

$$\boxed{\mathrm{E}\{\|\mathbf{H}-\hat{\mathbf{H}}_3(a_\mathrm{opt.})\|_F^2\} = \sigma^2\frac{N_\mathrm{R}\cdot N_\mathrm{T}}{K_\mathrm{P}+(1-2Pb)^2K_\mathrm{I}}}$$

  - asymptotically attains the CRLB at very low and high SNR

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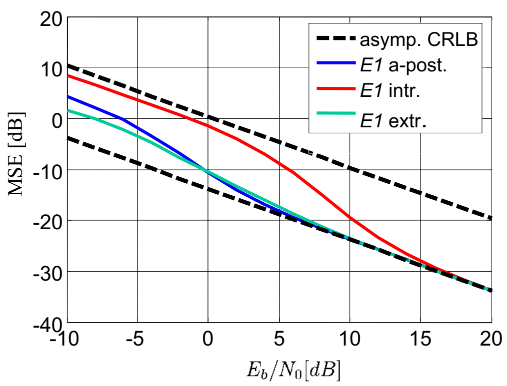
4.1 Numeric Results



- At low SNR *E1* exceeds CRLB due to its bias
- At high SNR *E1* always asymptotically meets the CRLB

- Simulation parameter: 4x2 block fading channel; BPSK; MAP equalizer and channel decoder; rate-1/4 repetition code; 5 iterations;

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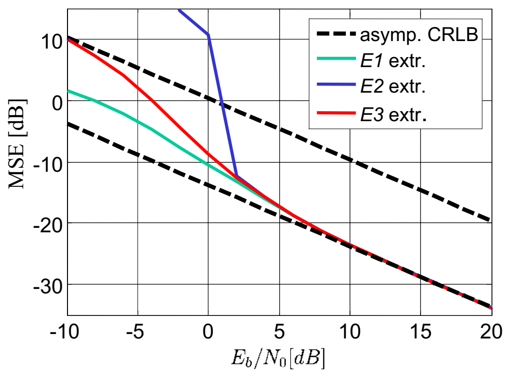
4.2 Numeric Results



- Since they are unbiased, the CRLB holds for *E2* and *E3*
- *E3* touches the asymptotic CRLB at low and high SNR

- Simulation parameter: 4x2 block fading channel; BPSK; MAP equalizer and channel decoder; rate-1/4 repetition code; 5 iterations;

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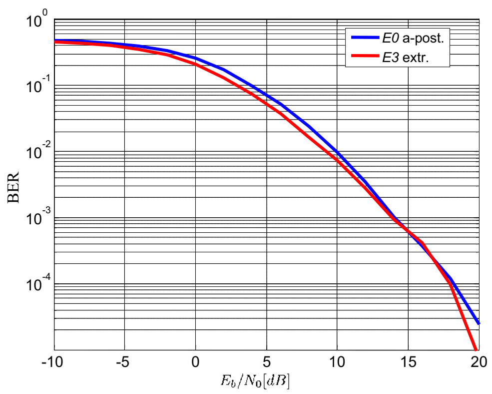
4.3 Numeric Results



- ☹ The influence of unbiased SBCE on the BER can be neglected
- ☹ Only for low code-rates a slight improvement can be observed

- Simulation parameter: 4x1 block fading channel; BPSK; MAP equalizer and channel decoder; rate-1/16 repetition code; 5 iterations;

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5. Conclusions

- Analysis of the MSE performance of SBCE with hard feedback
- Extrinsic feedback with appropriate reliability information can be used to obtain unbiased SBCE, e.g. *E2* or *E3*
- Asymptotical CRLB attainable, e.g. *E3*

- The influence of unbiased SBCE on the BER can be neglected

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Iterative Joint Channel Estimation and Decoding using Superimposed Pilots in OFDM-WLAN

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Introduction

- Approaches for Channel Estimation
  - Preamble-based Pilots (like 802.11a)
  - Superimposed Pilots (alternative)
- Requirements
  - Reliable Receiver Algorithms with Minimum Performance Degradation
  - Low Capacity Waste / Low Latency / Low Complexity
- Assumptions for IEEE802.11a-like Reference System under Investigation
  - All 64 subcarriers are available for data transmission
  - Preamble: Modified *Long Training Symbol* with equal probable ± 1 pilots through 64 subcarriers (*one OFDM symbol* in numerical examples)
  - Superimposed Pilots: Preamble above with appropriate power scaling
  - Channel Model ***h*** ($\boldsymbol{H} = \underline{\boldsymbol{F}}\boldsymbol{h}$): *Exponential Power Delay Profile* (max. CIR *L=8* taps, tap power decay $\propto$ *exp(-i)* at tap *i*, total channel power normalized to 1 in numerical examples)

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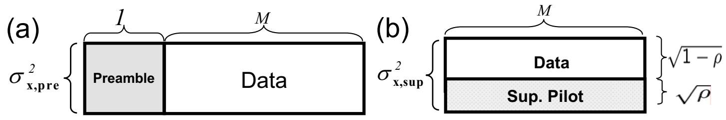
Comparison of Sup. Pilot and Preamble

- For a fair comparison, the Energy per information bit E_b should take account of the energy of the pilots and the preamble length and should be kept constant.



$$\left.\frac{E_b}{N_0}\right|_{sup} = \frac{1}{qR}\left(\frac{\sigma^2_{x,sup}}{\sigma^2_w}\right) = \frac{1}{qR}\left(\frac{\sigma^2_{x,pre}}{\sigma^2_w}\right)\cdot\frac{M+1}{M} = \left.\frac{E_b}{N_0}\right|_{pre}$$

- A QAM modulator with alphabet size $Q = 2^q$, the code rate R and Gaussian noise with variance σ^2_w

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Outline

- System Model and Receiver Architecture
- Channel Estimation for Superimposed Pilots
- Joint Channel Estimation and Decoding using Superimposed Pilots
- Conclusions

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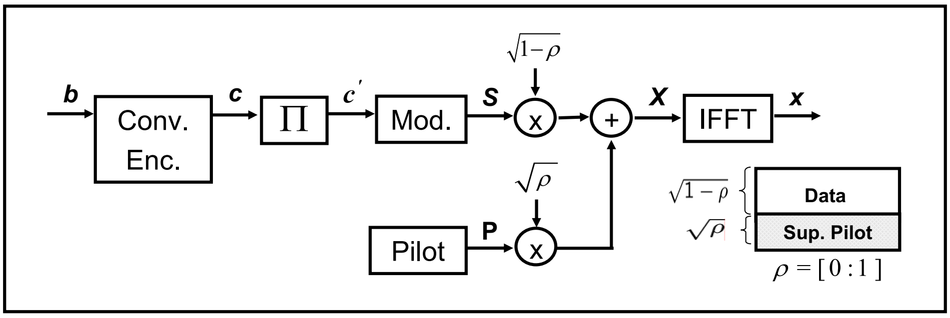
Transmitter with Superimposed Pilots



- Conv. Enc.: Non-Recursive Convolutional Code with Generator $G[7,5]_{oct}$
- Random interleaver
- Mod.: A Gray-mapping QAM modulator
- Guard interval same as IEEE802.11a

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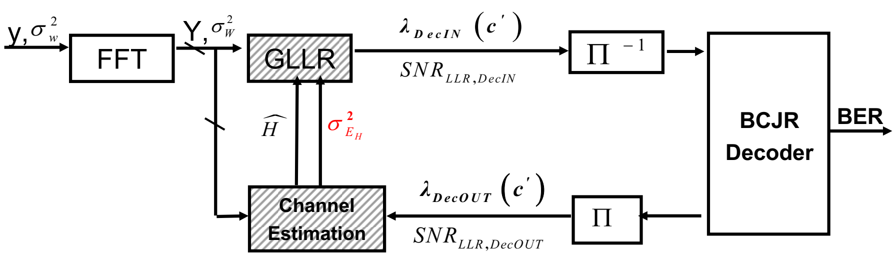
Joint Channel Estimation and Decoding

- Under appropriate conditions, channel and data estimates are refined in subsequent iterations until a convergence criterion or the maximum number of iterations is reached



- λ : Log-Likelihood Ratio
- SNR_{LLR} : SNR of Log-Likelihood Ratio
- GLLR: Generalized Log-Likelihood Ratio

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GLLR: Generalized Log-Likelihood Ratio

- Transmission model for a single subcarrier:

$$Y = \left(\sqrt{\rho}\,P + \sqrt{1-\rho}\,S\right)H + W\,, \qquad \sigma_P^2 = \sigma_S^2 = 1$$

$$\widehat{H} = H + E_H$$

- W and E_H are assumed to be independent, zero-mean, complex Gaussian random variables

- Based on Y and the variance σ_W^2 and $\sigma_{E_H}^2$, generalized LLRs of bit *k*: $\lambda_{DecIN}\left(c_k^{'}\right)$ for QAM leads to the following expression:

$$\lambda_{DecIN}\left(c_k^{'}\right) = \left(\min_{S^- \in \{S: c_k^{'}=1\}} \left\{ \frac{\left| Y - \left(\sqrt{\rho}P + \sqrt{1-\rho}S^-\right)\widehat{H}\right|^2}{\left|S^-\right|^2 \sigma_{E_H}^2 + \sigma_W^2} \right\} - \min_{S^+ \in \{S: c_k^{'}=0\}} \left\{ \frac{\left| Y - \left(\sqrt{\rho}P + \sqrt{1-\rho}S^+\right)\widehat{H}\right|^2}{\left|S^+\right|^2 \sigma_{E_H}^2 + \sigma_W^2} \right\}\right)$$

- QAM: Gray-Mapping
- How to estimate $\sigma_{E_H}^2$ will be discussed later

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Gaussian Density Model

- LLR values λs are assumed as i.i.d. and Gaussian distributed random variables with mean μ_λ and variance $\sigma_\lambda^2 = 2\mu_\lambda$

$$\lambda = \mu_\lambda(1 - 2C_k^{/}) + \sqrt{\sigma_\lambda^2}\,\mathcal{N}(0,1) \qquad C_k^{/} \in \{0,1\}$$

- "SNRs of Log-Likelihood Ratio" at decoder output or decoder input are defined as

$$SNR_{LLR} \triangleq \frac{\mu_\lambda^2}{\sigma_\lambda^2} = \frac{\mu_\lambda}{2}$$

- minimum $SNR_{LLR} = 10^{-3}$ in simulation

- We use *Gaussian density model* to characterize the performance of the iterative channel estimation and decoding algorithm and investigate the deviation by numerical simulation

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Estimate without *a-priori* Information during first Iteration

- Transmission model for channel estimation without *a-priori* information during first iteration

$$Y = \left(\boxed{\sqrt{\rho}\,\underline{P}\,\underline{F}}\,h\right) + \left(\boxed{\sqrt{1-\rho}\,\underline{S}H + W}\right)$$

Effective Pilot — $\underline{X}_{eff}$

Effective Noise — $\underline{W}_{eff}$ with variance $\sigma_{eff}^2 = \left[(1-\rho)\sigma_{X,sup}^2 + \sigma_W^2\right] / M$

- Least-square channel estimation with superimposed pilots
  - Estimator: $\widehat{H} = \underline{F}\hat{h} = \underline{F}\left[\left(\underline{X}_{eff}{}^H \underline{X}_{eff}\right)^{-1} \underline{X}_{eff}{}^H Y\right]$
  - Channel estimation error: $\sigma_{E_H,LS}^2 = \frac{L}{N\rho}\sigma_{eff}^2$
- MMSE channel estimation (see our paper)
- Variance of channel estimate found by simulation agrees exactly with estimation theory

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Estimate with *a-priori* Information at subsequent Iterations

- Transmission model for channel estimation with *a-priori* information at subsequent iterations

$$Y = \left(\boxed{\sqrt{\rho}\,\underline{P} + \sqrt{1-\rho}\,\underline{\widehat{S}}\right)\underline{F}}\,h + \boxed{\sqrt{1-\rho}\left(\underline{S} - \underline{\widehat{S}}\right)H + W}$$

Effective Pilot — $\underline{X}_{eff}$ (random variables)

Effective Noise — $\underline{W}_{eff}$ with variance $\sigma_{eff}^2 = \left[(1-\rho)\sigma_{E_S}^2 + \sigma_W^2\right] / M$

- Symbol estimation error ($E_S = S - \widehat{S}$) is simulated by applying *Gaussian Density Model*, *soft or hard decision* and *a Gray-mapping QAM modulator* to randomly generated source bits with given $SNR_{LLR,DecOUT}$
- *Channel estimation error* is investigated by replacing a preamble as effective pilot and Gaussian noise as effective noise in preamble-based estimation theory, if *SNR of Log-Likelihood Ratio* is known
- Channel estimation theory under present assumptions matches the real case by simulation well at high $SNR_{LLR,DecOUT}$

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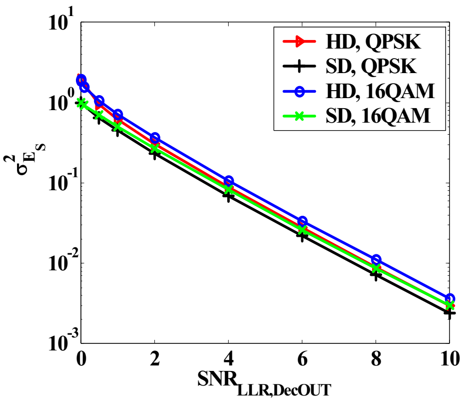
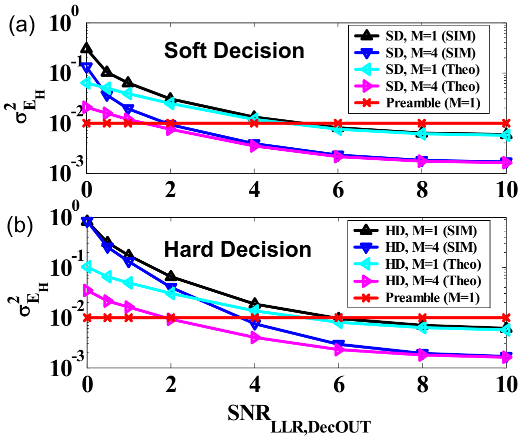
Channel Estimation with *both* Pilot and Decoded Data

- Channel estimation theory under present assumptions matches the real case by simulation well at high $SNR_{LLR,DecOUT}$





Symbol Estimation Error Variance: one OFDM symbol

MMSE Channel Estimator: Eb/N0=10dB, 16 QAM (Gray-Mapping), Exponential Power Delay Profile, Sup. Pilot ($\rho = 0.1$)

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Iterative Channel Estimation and Decoding Algorithm

- The estimation error variance $\sigma^2_{E_H}$ is required by GLLR at receiver using superimposed pilots
- The variance $\sigma^2_{E_H}$ in the first iteration can be exactly determined by theory
- In the subsequent iterations, a practical receiver can be implemented as follows
  - Estimate " $SNR_{LLR,DecOUT}$ " at input of channel estimator,
  - For given $SNR_{LLR,DecOUT}$,calculate the symbol estimation error variance $\sigma^2_{E_S}$,
  - Calculate theoretical $\sigma^2_{E_H}$

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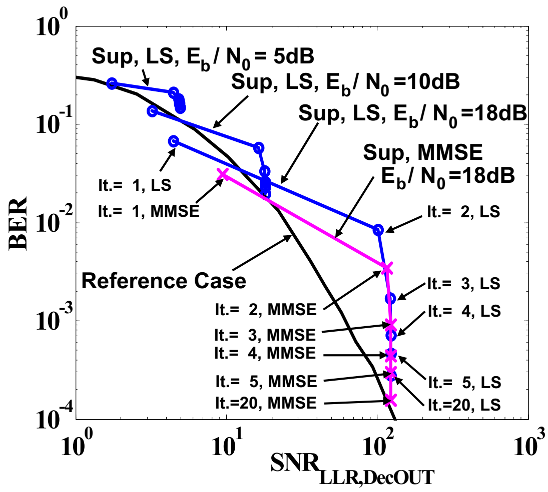
Unique Mapping of $SNR_{LLR,DecOUT}$ vs. BER regardless of Pilot Design



- Observation of convergence behaviour (BER vs. $SNR_{LLR,DecOUT}$ instead of normal SNR)
- Over- / under-estimated $SNR_{LLR,DecOUT}$ at decoder output converges to approximate perfect estimate of the preamble technique by iterations
- Corrected $SNR_{LLR,DecIN}$ and $SNR_{LLR,DecOUT}$ are used to track the density evolution of LLR values

Sup. Pilot (M=4, $\rho=0.15$), 16QAM (Gray-Mapping), Exponential Power Delay Profile

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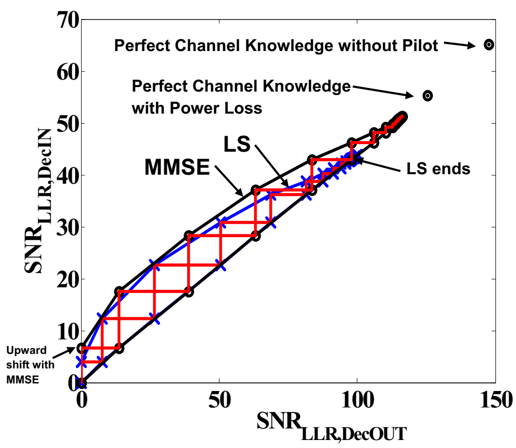
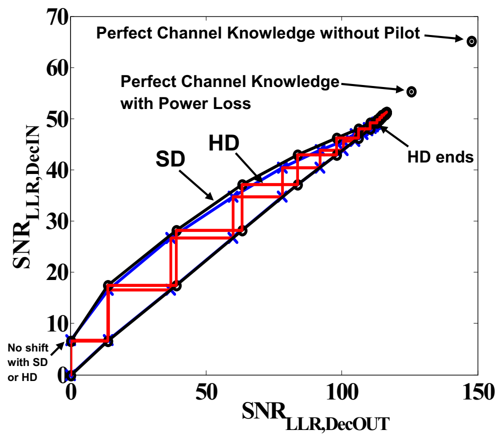
LS / MMSE Channel Estimator with SD or HD

- MMSE improve both initial and final SNR_{LLR}
- SD improves the final SNR_{LLR} , but not dramatically





Sup. Pilot (M=4, $\rho=0.15$, iter.=20), Exponential Power Delay Profile, $E_b/N_0=18dB$, 16QAM (Gray-Mapping),

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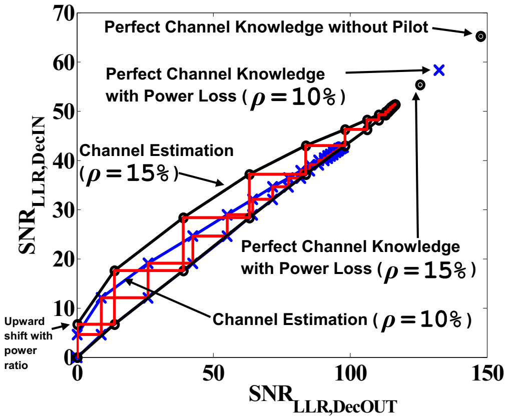
Effect of Power Ratio



- If $\rho\downarrow$, decoder performance with perfect channel knowledge $\uparrow$ but channel estimator performance $\downarrow \Rightarrow$ aggregate performance $\downarrow$ in this case.
- In general: optimal ρ exists

Sup. Pilot (M=4, $\rho=0.15$, iter.=20), Exponential Power Delay Profile, $E_b / N_0 = 18dB$, 16QAM (Gray-Mapping), MMSE Channel Estimator with SD

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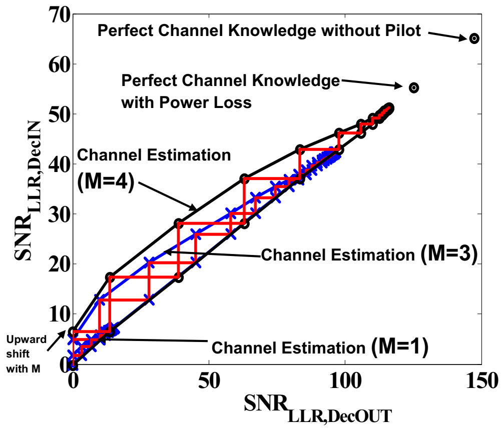
Effect of Packet Length



- M $\uparrow$ (power ratio fixed) $\Rightarrow$ aggregate performance $\uparrow$, because the quality of channel estimate $\uparrow$ by averaging more symbols

Sup. Pilot (M=4, $\rho=0.15$, iter.=20), Exponential Power Delay Profile, $E_b / N_0 = 18dB$, 16QAM (Gray-Mapping), MMSE Channel Estimator with SD

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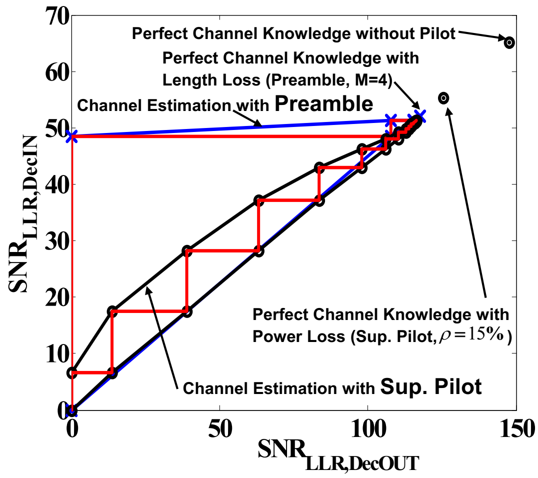
Preamble vs. Sup. Pilot Techniques



- Condition: $\left.\frac{E_b}{N_0}\right|_{sup} = \left.\frac{E_b}{N_0}\right|_{pre}$
- A preamble technique converges after 3 iterations, whereas a Sup. Pilot technique after at least 10 iterations
- As M decreases, performance of a preamble technique decreases monotonously, because relatively more energy is invested in estimating channel

Sup. Pilot (M=4, $\rho = 0.15$, iter.=20), Exponential Power Delay Profile, $E_b / N_0 = 18dB$, 16QAM (Gray-Mapping), MMSE Channel Estimator with SD

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Optimal System Parameters (QPSK)

- Dark shaded areas indicate the optimal range of system parameters determined by sampled simulation ($\rho < \frac{1}{M+1}$) under $\left.\frac{E_b}{N_0}\right|_{sup} = \left.\frac{E_b}{N_0}\right|_{pre}$

M \ ρ	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10
1	-0.54 / -0.48	-0.47 / -0.44	-0.20 / -0.17	-0.16 / -0.10	-0.41 / -0.35	-0.92 / -0.82	-3.35 / -3.21	-11.1 / -10.8	-39.0 / -38.6
2					-0.02 / -0.01	0.18 / 0.23	0.04 / 0.11	-0.43 / -0.30	-8.70 / -8.43
3						-0.10 / -0.08	0.08 / 0.08	0.17 / 0.17	-1.58 / -1.26
4							-0.10 / -0.08	0.01 / 0.03	-0.26 / -0.18
5								0.04 / 0.04	0.16 / 0.18

- **(x / y) = (iter.=10 / iter.=20) for Sup. Pilots**
- **and x or y =** $(BER_{\text{pre}} - BER_{\text{sup}})/BER_{\text{pre}}$

Exponential Power Delay Profile, $E_b / N_0 = 16dB$, MMSE Channel Estimator with Soft Decision, Preamble (iter. = 3)

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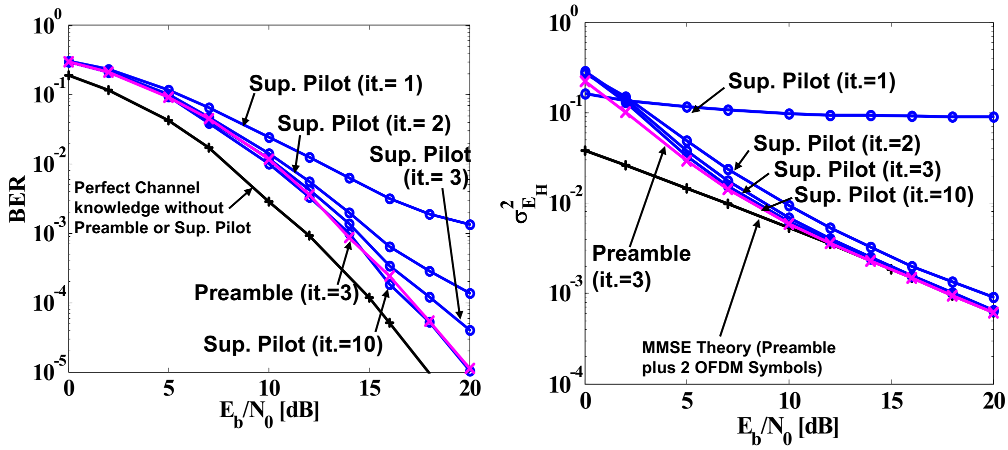
Iterative Behavior (QPSK)

- Sup. pilot technique achieves almost the same performance as a preamble technique by iterative algorithms through all E_b/N_0



Sup. Pilot (M=2, $\rho=0.25$), Exponential Power Delay Profile , MMSE Channel Estimator with Soft Decision

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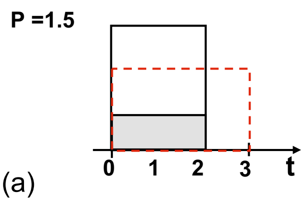
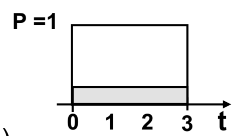
Applications in WIGWAM

- QPSK with optimized system parameters (M=2, $\rho=0.25$) in uplink



(a)



(b)

- Transmission delay shortened
- Same *average power* and *energy per information bit*
- Same BER performance

- Throughput increases
- Same *transmission power* and one more *data symbol* sent
- Negligible BER loss

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Conclusion

- A practical joint channel estimation and data detection algorithm using Sup. Pilot is proposed. Its iterative characteristics and performance are investigated and optimized
- Sup. Pilot technique and preamble technique have similar BER performance through all Eb/N0 under optimized power ratio and data packet length (assumptions: MMSE channel estimator with Soft Decision) with limited complexity (iteration ≦ 10)
- Sup. Pilot technique is suitable and flexible for systems including high percentage of short bursty data traffic with lower QAM (such as QPSK) to reduce transmission latency or increase throughput

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LEHRSTUHL FÜR NACHRICHTENTECHNIK
Prof. Dr.-Ing. R. Urbansky

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DIORAMA - An Iterative Decoding Real-Time MATLAB Receiver for the Multicarrier-Based Digital Radio DRM

A. Dittrich, T. Schorr, W. Sauer-Greff, R. Urbansky
University of Kaiserslautern, Institute of Communications Engineering

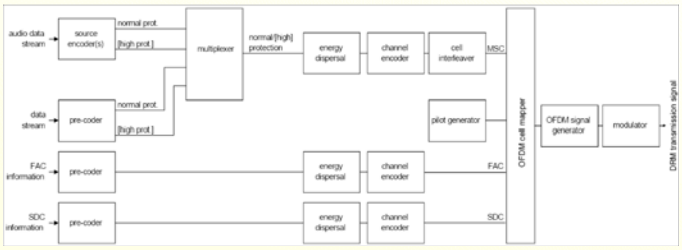
6. Sitzung der ITG Fachgruppe "Angewandte Informationstheorie"
Kaiserslautern, October 7, 2005

Outline

- Digital Radio Mondiale (DRM): System Overview
  - Source Coding
  - Channel Coding
  - Transmission Structure
- DIORAMA - A PC Based Real-Time MATLAB Receiver
  - Demodulation, Synchronization and Equalization
  - Iterative Channel Decoding
  - Source Decoding, Multimedia Output
- Summary
- Online Demonstration

Digital Radio Mondiale (DRM)

Digital Audio Broadcasting up to 30 MHz RF-Carrier



❍ Source Coding

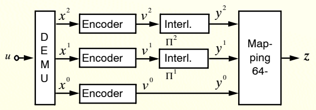
MPEG-4 AAC HE / CELP / HVXC
SBR (Spectral Band Replication), ER (Error Resilence), PS (Parametric Stereo)
Logical channels: FAC, SDC, MSC

❍ Transmission Modes

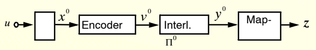
Channel spacing: 9 / 10 kHz; 4.5 / 5 kHz; [18 / 20] kHz
4 Robustness Modes; 4/16/64-QAM; OFDM (288/256/176/112 carriers), cyclic prefix
Convolutional coding R=0.48...0.78, Unequal Error Protection, 2 seconds interleaving
Scattered pilot cells for frequency and timing synchronization

Channel Coding

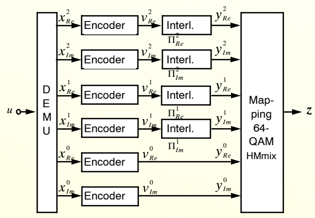
- Multi-Level Coding (MLC) adaptable for different transmission scenarios and conditions
- Ungerboeck set partitioning for standard mapping, block partitioning for hierarchical mapping or a mixture of both:



64-QAM ML Coding SM or HM

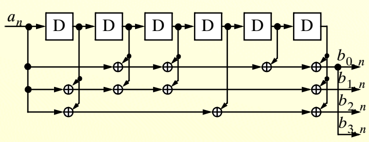


4-QAM ML Coding (FAC, SDC)



64-QAM ML Coding SM and HM (HMmix)

- Constituent convolutional codes for Multi-Level Coding (MLC)
- Rates from 1/4 to 8/9 by puncturing of a rate 1/4 convolutional mother code:



OFDM Symbol Parameters

4 Robustness Modes for different transmission scenarios

Mode	**A**	**B**	**C**	**D**
OFDM Symbol Duration T_s/ms, (T_s/T)	26.66 (320)	26.66 (320)	20.00 (240)	16.66 (200)
Guard Interval Duration T_g/ms, (T_s/T)	2.66 (32)	5.33 (64)	5.33 (64)	7.33 (88)
Max. Echo Path Length	800 km	1600 km	1600 km	2200 km
FFT-size	288	256	176	112
T_g/T_u	1/9	1/4	4/11	11/14
Carrier Spacing Δf/Hz	41.66	46.88	68.18	107.14
Number of Carriers @ 9 kHz K	204	182	---	---
Number of Carriers @ 10 kHz K	228	206	138	88
Number of Carriers @ 18 kHz K	412	366	---	---
Number of Carriers @ 20 kHz K	460	410	280	178
OFDM Symbols per Frame N_s	15	15	20	24
MSC Net Data Rate in kbit/s	6.2 - 71.9	4.8 - 56.1	9.1 - 45.4	6.0 - 30.6

Pilot Distribution

```
symbols (s)                                  DC
                 negative frequencies         :        positive frequencies
   1..1                                        :                                        1..1
   1..0..33333333322222222221111111111000000000:00000000011111111112222222222333333333..0..1
   4..2..87654321098765432109876543210987654321:12345678901234567890123456789012345678..2..4
                                               :
0  .  .  O...................O................:::O...................O................  O  .
1  O  .  ....O...................O............:::....O...................O............  .  .
2  .  .  ........O...................O........:::........O...................O........  .  .
3  .  .  ............O...................O....:::............O...................O....  .  O
4  .  O  ................O...................O:::................O...................O  O  .
5  .  .  O...................O................:::O...................O................  O  .
6  O  .  ....O...................O............:::....O...................O............  .  .
7  .  .  ........O...................O........:::........O...................O........  .  .
8  .  .  ............O...................O....:::............O...................O....  .  O
9  .  O  ................O...................O:::................O...................O  O  .
10 .  .  O...................O................:::O...................O................  O  .
11 O  .  ....O...................O............:::....O...................O............  .  .
12 .  .  ........O...................O........:::........O...................O........  .  .
13 .  .  ............O...................O....:::............O...................O....  .  O
14 .  O  ................O...................O:::................O...................O  O  .
```

Mode A

```
symbols (s)                                  DC
                 negative frequencies         :        positive frequencies
                                               :
   44444333333333322222222221111111111000000000:00000000011111111112222222222333333333344444
   43210987654321098765432109876543210987654321:12345678901234567890123456789012345678901234
                                               :
0  O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.:O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.
1  .O..O..O..O..O..O..O..O..O..O..O..O..O..O..O:.O..O..O..O..O..O..O..O..O..O..O..O..O..O..O
2  ..O..O..O..O..O..O..O..O..O..O..O..O..O..O..:..O..O..O..O..O..O..O..O..O..O..O..O..O..O..
3  O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.:O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.
4  .O..O..O..O..O..O..O..O..O..O..O..O..O..O..O:.O..O..O..O..O..O..O..O..O..O..O..O..O..O..O
5  ..O..O..O..O..O..O..O..O..O..O..O..O..O..O..:..O..O..O..O..O..O..O..O..O..O..O..O..O..O..
6  O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.:O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.
7  .O..O..O..O..O..O..O..O..O..O..O..O..O..O..O:.O..O..O..O..O..O..O..O..O..O..O..O..O..O..O
8  ..O..O..O..O..O..O..O..O..O..O..O..O..O..O..:..O..O..O..O..O..O..O..O..O..O..O..O..O..O..
9  O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.:O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.
10 .O..O..O..O..O..O..O..O..O..O..O..O..O..O..O:.O..O..O..O..O..O..O..O..O..O..O..O..O..O..O
11 ..O..O..O..O..O..O..O..O..O..O..O..O..O..O..:..O..O..O..O..O..O..O..O..O..O..O..O..O..O..
12 O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.:O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.
13 .O..O..O..O..O..O..O..O..O..O..O..O..O..O..O:.O..O..O..O..O..O..O..O..O..O..O..O..O..O..O
14 ..O..O..O..O..O..O..O..O..O..O..O..O..O..O..:..O..O..O..O..O..O..O..O..O..O..O..O..O..O..
15 O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.:O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.
16 .O..O..O..O..O..O..O..O..O..O..O..O..O..O..O:.O..O..O..O..O..O..O..O..O..O..O..O..O..O..O
17 ..O..O..O..O..O..O..O..O..O..O..O..O..O..O..:..O..O..O..O..O..O..O..O..O..O..O..O..O..O..
18 O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.:O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.
19 .O..O..O..O..O..O..O..O..O..O..O..O..O..O..O:.O..O..O..O..O..O..O..O..O..O..O..O..O..O..O
20 ..O..O..O..O..O..O..O..O..O..O..O..O..O..O..:..O..O..O..O..O..O..O..O..O..O..O..O..O..O..
21 O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.:O..O..O..O..O..O..O..O..O..O..O..O..O..O..O.
22 .O..O..O..O..O..O..O..O..O..O..O..O..O..O..O:.O..O..O..O..O..O..O..O..O..O..O..O..O..O..O
23 ..O..O..O..O..O..O..O..O..O..O..O..O..O..O..:..O..O..O..O..O..O..O..O..O..O..O..O..O..O..
```

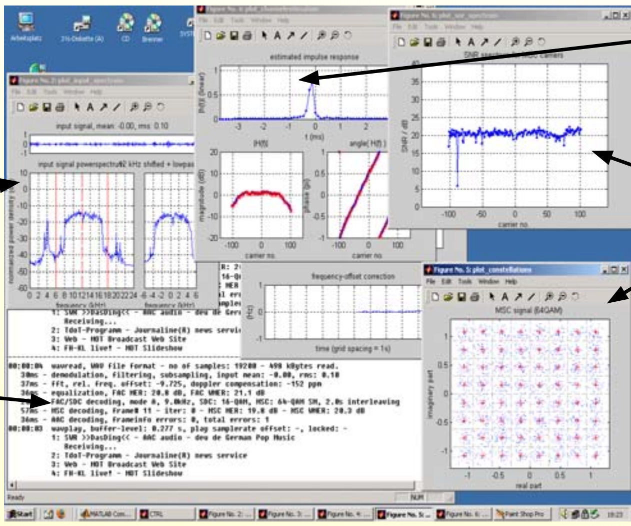
Mode D

DIORAMA - DRM Receiver for MATLAB

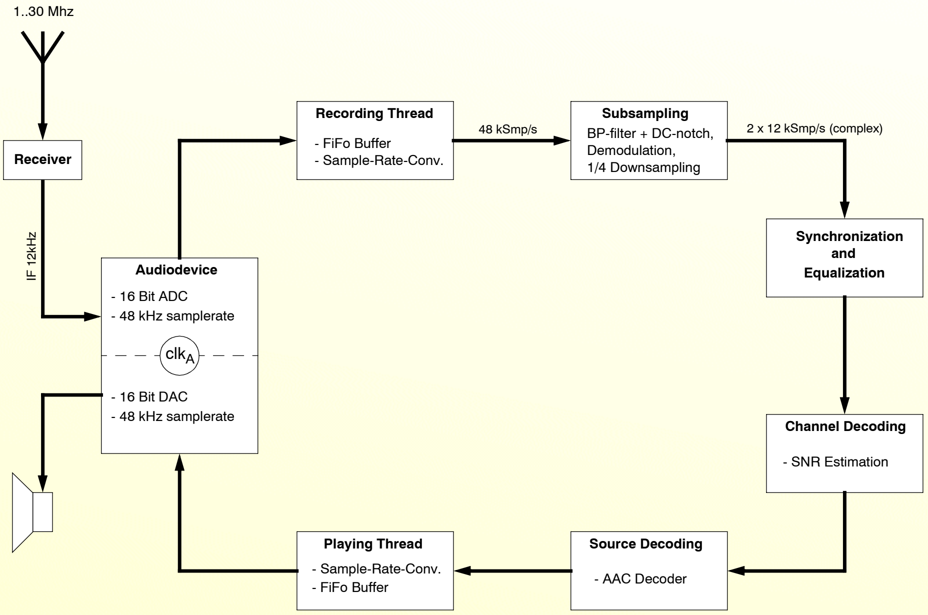
- Greek "dia" for "through", "horan" for "look"
  Developed for Educational Purpose to look "through" the surface of a software DRM receiver
- Real-time radio including text message decoding, Journaline(R) news service, web page broadcasting
- Optional Graphical User Interface
- Online display of input spectrum, synchronization variables, channel estimation, constellations and SNR at each carrier...
- Most variables real-time accessible in MATLAB workspace

Screenshot

channel estimation
input spectrum
SNR-spectrum
constellation
MATLAB command window
(program flow and
signal processing info)



Signal Processing: Overview



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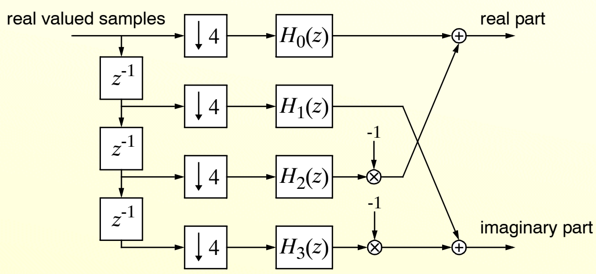
Recording and Subsampling

- Polyphase dynamic sampling rate conversion for received signal (input) and audio signal (output)
- Combined filtering, demodulation and subsampling

intermediate frequency, 48 kSmp/s **baseband, 2 x 12 kSmp/s**



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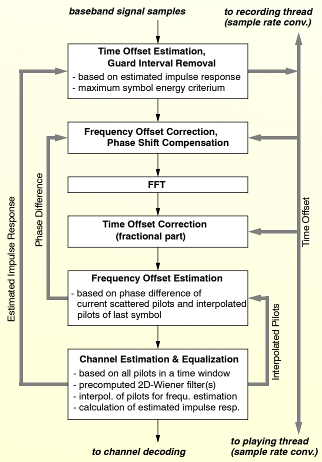
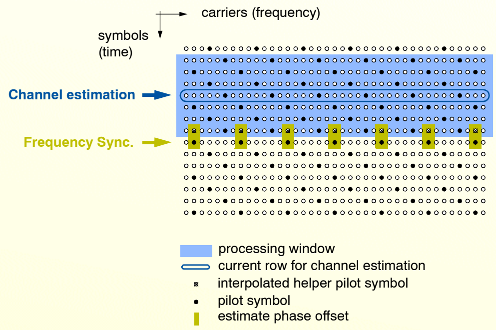
Synchronization and Equalization

Processing Flow (Tracking)

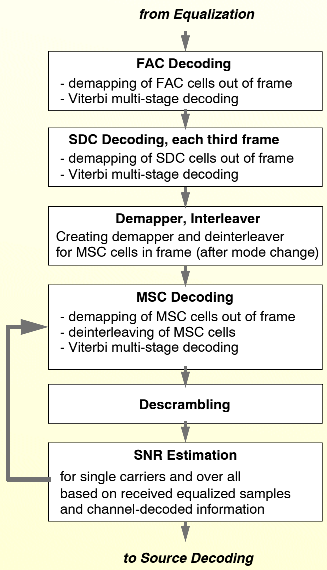


2-D-Wiener Filter

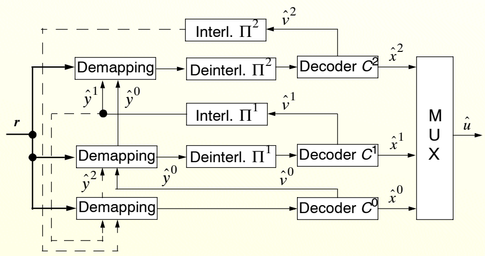


Channel Decoding (I)

Processing Flow



Multi-Stage-Decoder



- Constituent Viterbi Decoders
- Iterative Decoding
- SNR Estimation based on channel transfer function, decoded data and equalized OFDM cells
- Signal Histogram
- Data descrambling and distribution to services

Channel Decoding (II)

- SNR estimation useful for Viterbi decoding
- SNR depending on channel transfer function and noise power density
- Channel transfer function: fast changing; noise power density: slowly changing => seperate estimation of channel transfer funtion and noise power density and calculation of SNR

Optimal metric for optimal codes:

$$L(\hat{y}_n^{\,j}) = \frac{\hat{h}_n^2}{\sigma_n^2}\left|\frac{r_n}{\hat{h}_n} - z_n^0\right|^2 - \frac{\hat{h}_n^2}{\sigma_n^2}\left|\frac{r_n}{\hat{h}_n} - z_n^1\right|^2$$

Better metric for non-optimal codes:

$$L(\hat{y}_n^{\,j}) = \frac{\hat{h}_n}{\sigma_n} \cdot \left|\frac{r_n}{\hat{h}_n} - z_n^0\right| - \frac{\hat{h}_n}{\sigma_n} \cdot \left|\frac{r_n}{\hat{h}_n} - z_n^1\right|$$

Noise variance:

$$\sigma_n^2 = \mathrm{E}\left\{\left|r_n - \hat{z}_n \hat{h}_n\right|^2\right\}$$

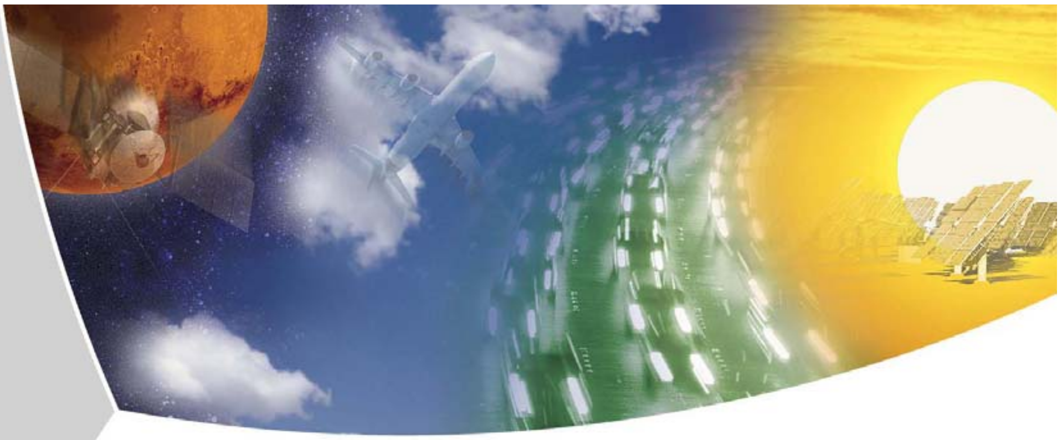
- Interleaving delay can be reduced starting decoding before all samples of a code block are received: Missing samples are marked as erasures by setting their h_n to 0.

Source Decoding

- AAC decoding supporting
  Equal and Unequal Error Protection (UEP),
  (parametric) Stereo, Spectral Band Replication (SBR)
- Data unit and file assembling for data reception
- Deflate uncompression for compressed text
- News Service Journaline(R) decoding

Summary

- Diorama is a complete open source software radio receiver for Digital Radio Mondiale (DRM), based on MATLAB
- Valuable for research and teaching:
  Running in real-time on MATLAB, it offers the interested user many features for educational and research purposes.
- Easy debugging and development:
  The decoding process is interruptible at each time and the MATLAB debugger can be used for verification and visualization.



Radio Resource Management for MC-CDMA over Correlated Rayleigh Fading Channels

Simon Plass and Armin Dammann

German Aerospace Center (DLR)

Kaiserslautern, October 7, 2005

German Aerospace Center

Outline

- MC-CDMA Downlink System
- Multi-Cell Environment
- Correlated Rayleigh Fading Channel
- Radio Resource Management with Spreading Codes
- Simulation Results and Conclusions

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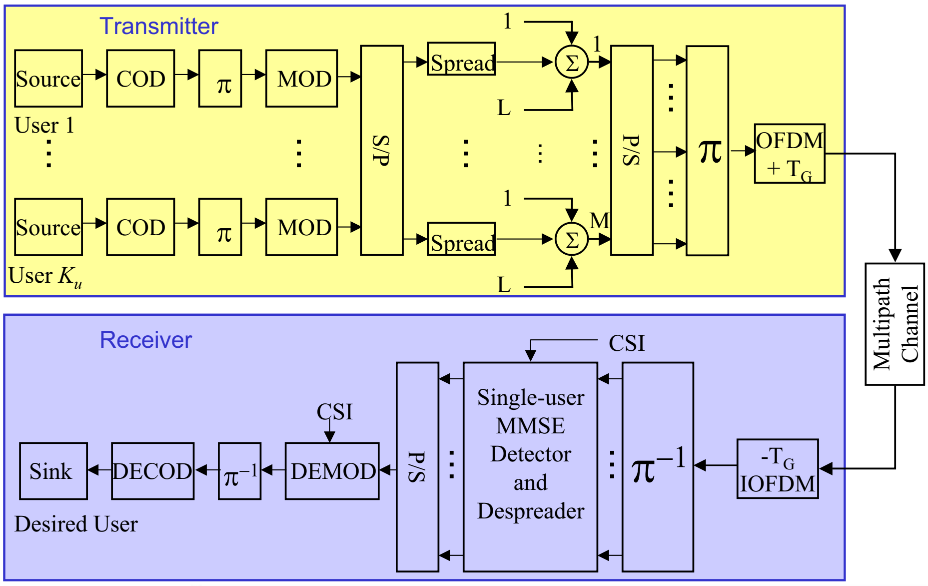
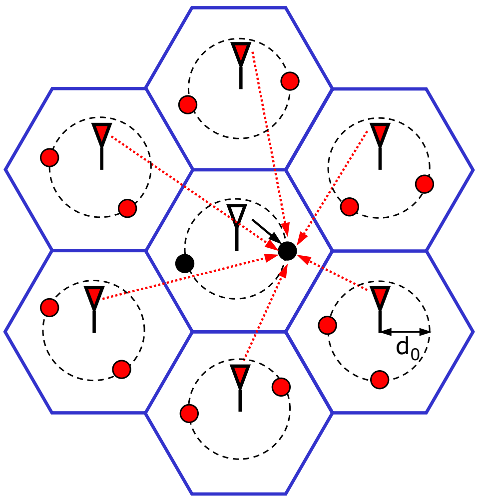
MC-CDMA Downlink System

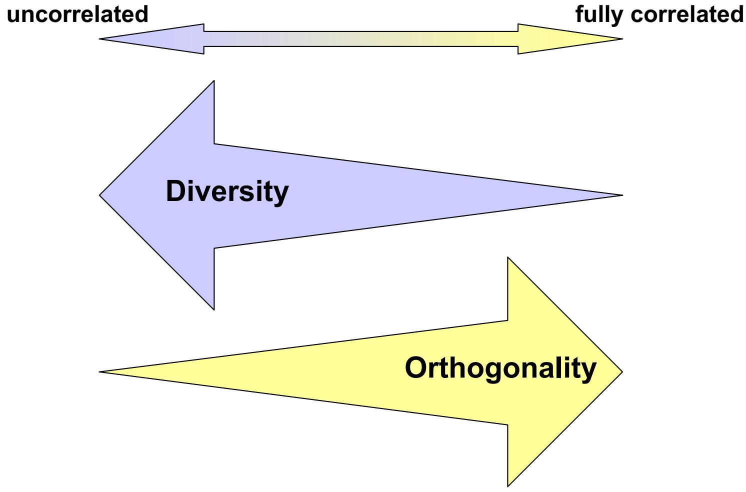


Multi-Cell Environment

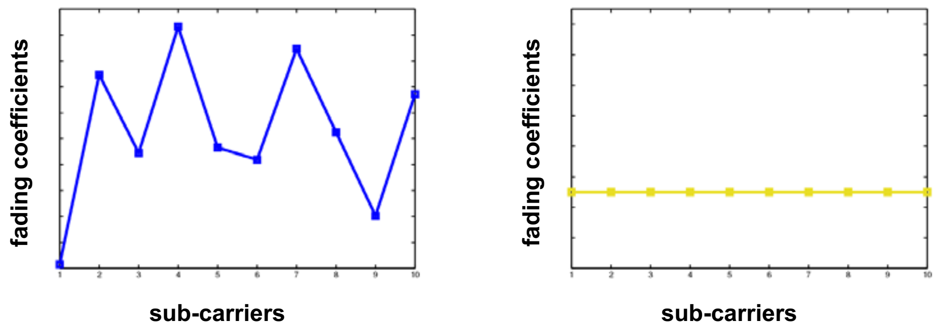


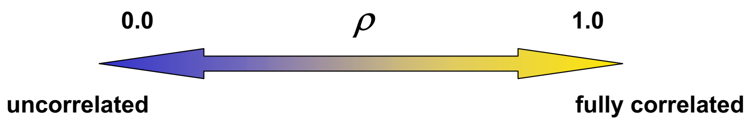
- Fully-synchronized system between the base stations (BS) and the mobile station (MS)
- Distance dependent propagation model
- All users are power-controlled and are located at a distance d_0 to their BSs
- Cells are equally loaded

Correlation Properties - Contrariness of Diversity and Orthogonality

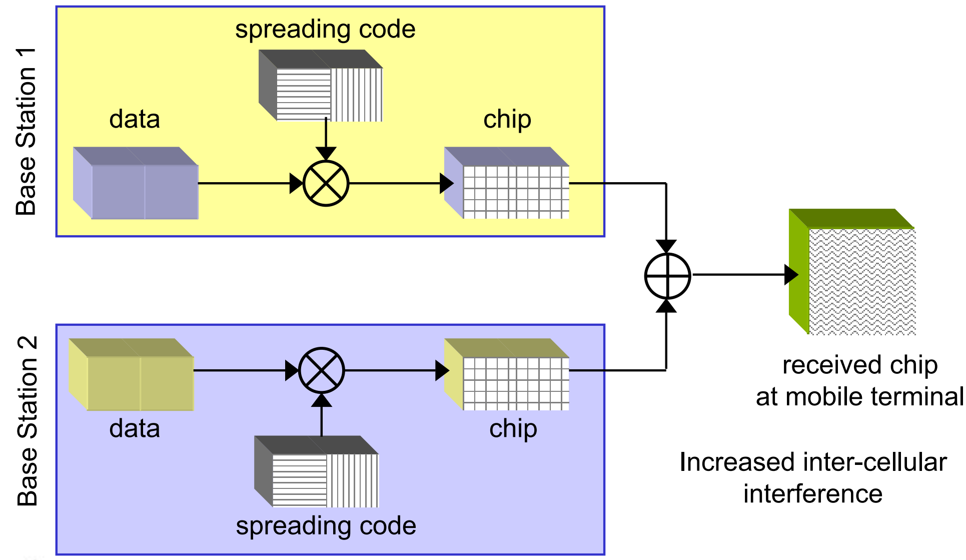


Correlated Rayleigh Fading Channel

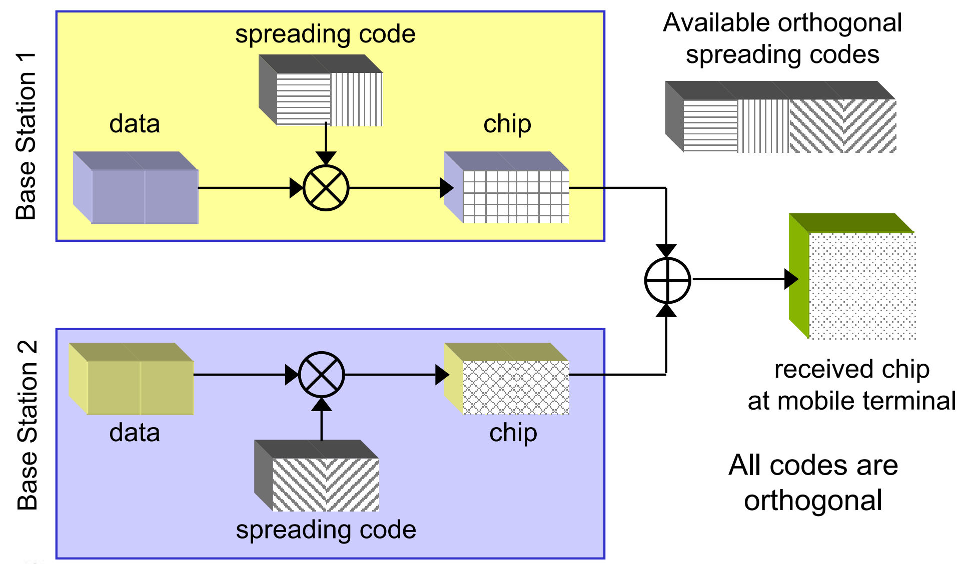




Without Radio Resource Management

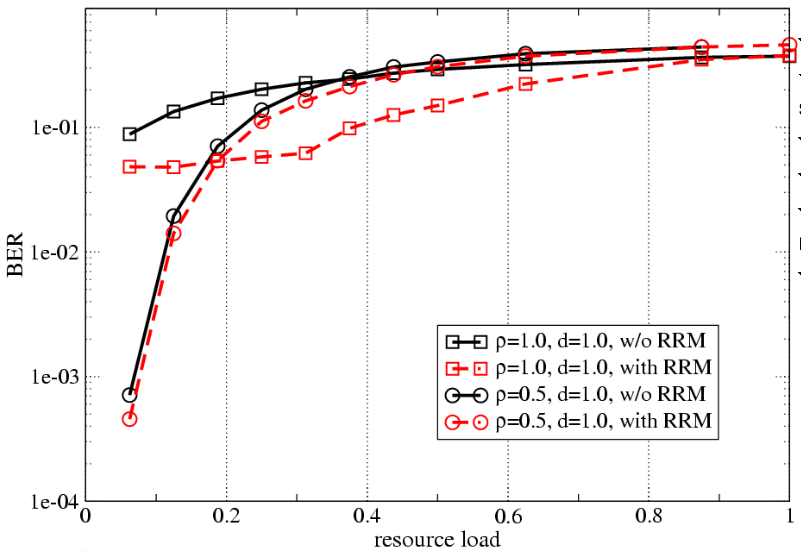


Radio Resource Management with Spreading Codes



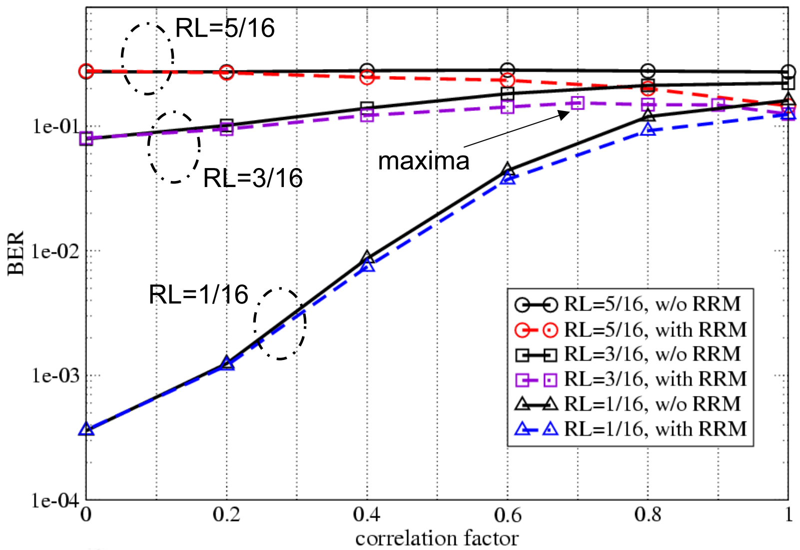
Simulation Results

Influence of Resource Load



- Interfering cells =6
- Perfect channel state information
- QPSK
- L = 16
- Conv. code with rate ½
- E_b/N_0 = 10 dB

Influence of Correlation



- Interfering cells =6
- Perfect channel state information
- QPSK
- L = 16
- Conv. code with rate ½
- E_b/N_0 = 10 dB

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11

Conclusions

- Trade-off between diversity and orthogonality due to the correlation properties of the Rayleigh fading channel
- Possibility of using an RRM over spreading codes because a multi-path propagation environment has a degree of correlation

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12

Concepts for Accurate Low-Cost Signature Based Localisation of Mobile Terminals

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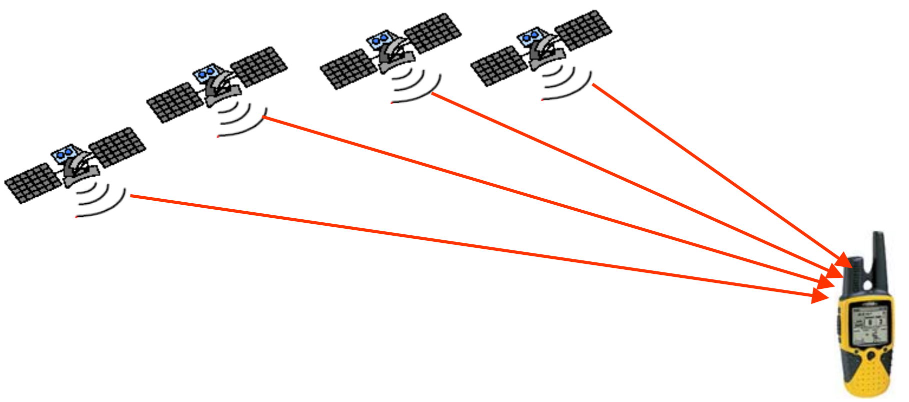
Research Group for RF Communications, University of Kaiserslautern

email: sheilman@rhrk.uni-kl.de

obvious approach

- satellite based localization schemes (GPS, Galileo, GLONASS)



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objectives

HS

- localization in scenarios, in which satellite based systems do not work (autonomous localization schemes)
- combination of autonomous localization schemes with satellite based localization schemes (A-GPS)

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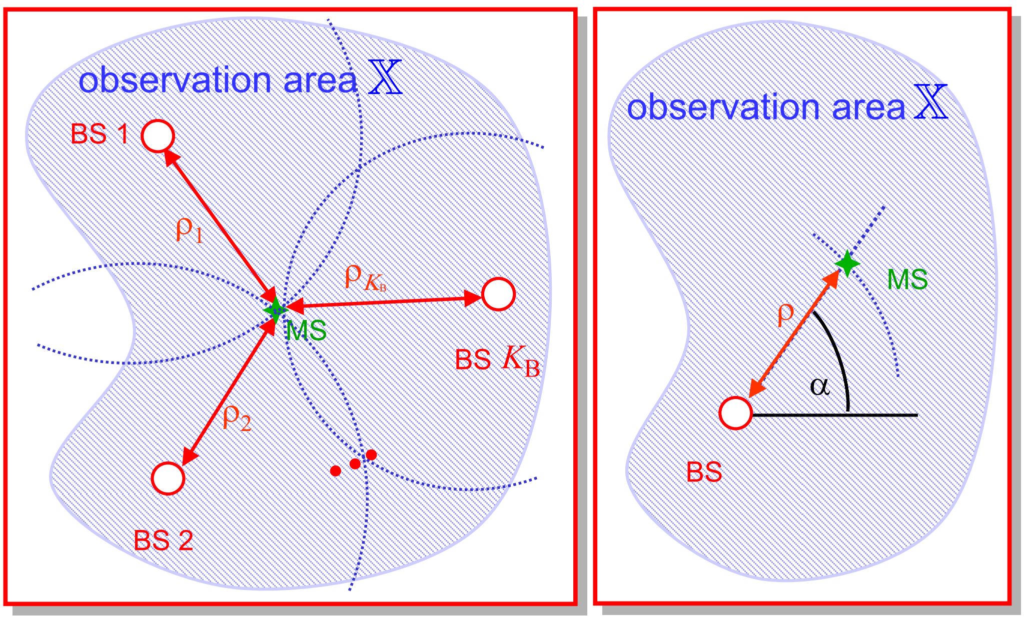
classic autonomous geometry based localization

HS



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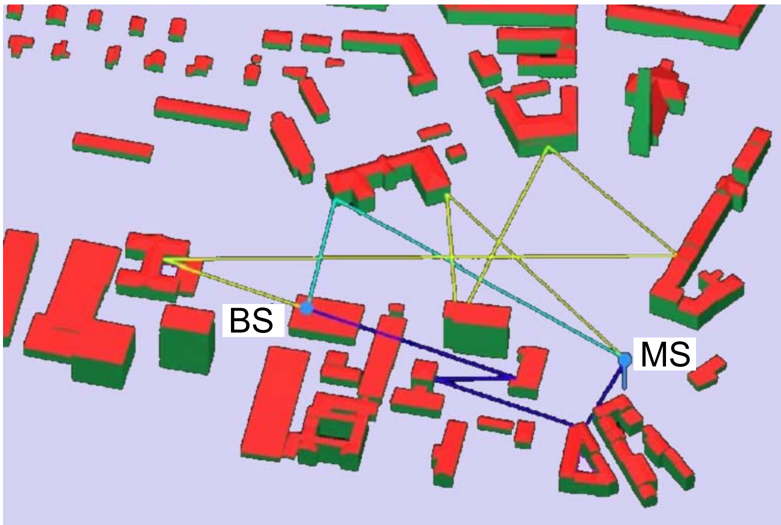
multipath scene

- multipath-scattering
- no optical line of sight



source: J. Maurer, W. Wiesbeck et al., University of Karlsruhe (TH)

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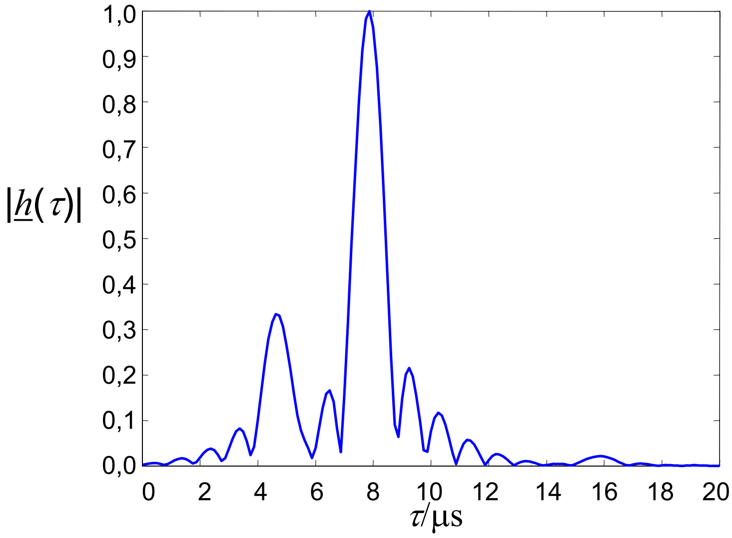
full CIR between BS und MS



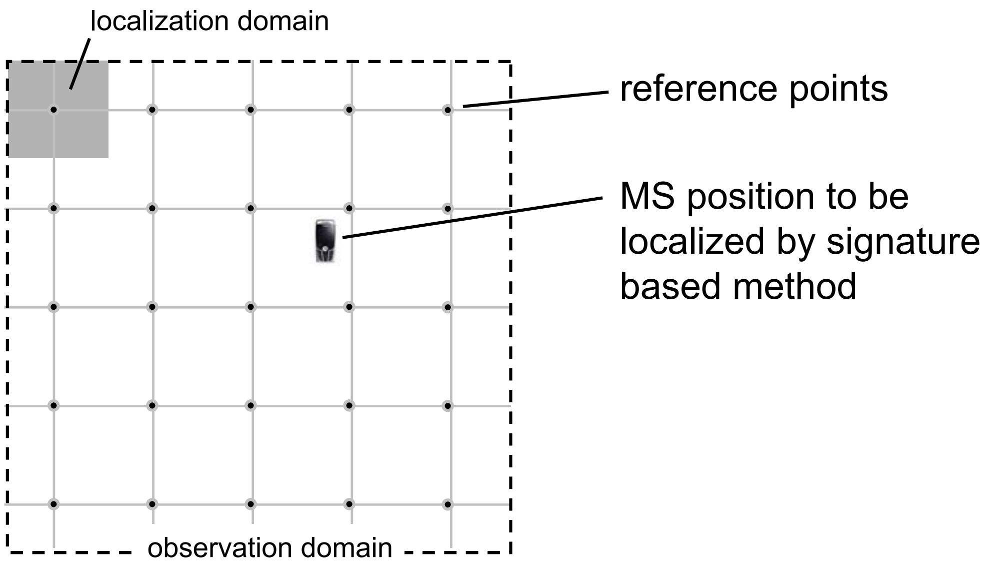
channel impulse response (CIR) is a nearly unique characteristic (signature) for the location of an MS

signature based localization: principle

offline:
- store signatures for sufficiently many reference points of the scenario in a data base

online:
- comparison of currently measured signature with entries in data base
- „best match“ gives the most probable position of the MS

signature based localization: observation domain



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choice of signature: CIR

HS

CIR changes for small movements in order of $\lambda/2$ due to fast fading

very small RP spacing required

prohibitively big database

??? how to decrease database size ???

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localization using covariance matrices (CMs)

HS

offline determined and stored in database for each RP k_r

comparison for each RP k_r

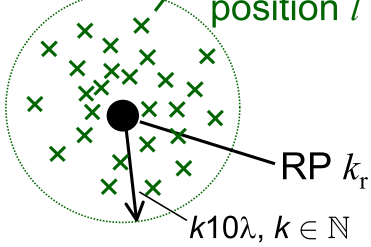
online measured

$$\underline{\mathbf{R}}^{(k_r)} = \mathrm{E}\left\{\underbrace{\underline{\mathbf{h}}_l \underline{\mathbf{h}}_l}^{\mathrm{H}}\right\}$$

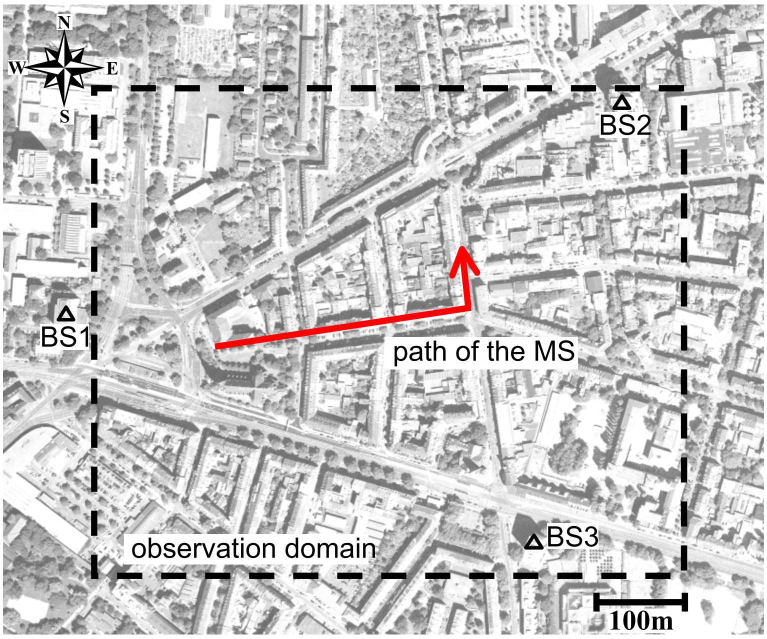
$$\underline{\mathbf{R}}_{\mathrm{MS}} = \mathrm{E}\left\{\underbrace{\underline{\mathbf{h}}_l \underline{\mathbf{h}}_l}^{\mathrm{H}}\right\}$$

measurement position l

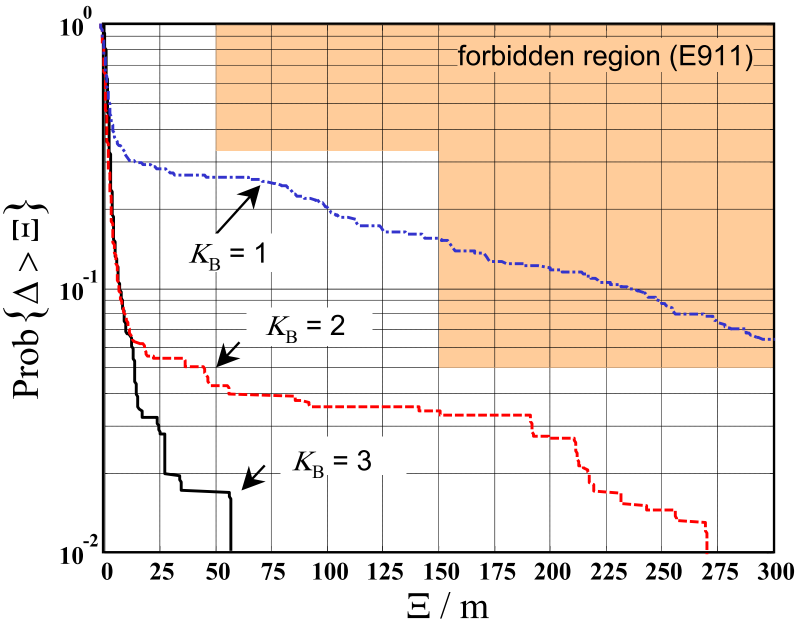
MS position l



exemplary scenario: downtown Karlsruhe



CCDF of localization error Δ: parameter number of BSs K_B



scenario:

- $K_B = 1 \ldots 3$
- $\gamma \to \infty$

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conclusions

- CMs of CIRs suited as signatures for MS localization
- CMs are especially unique in multi-path non line-of-sight (NLOS) scenarios
- achievable localization accuracy quite promising even if considering a single BS

LEHRSTUHL FÜR NACHRICHTENTECHNIK
Prof. Dr.-Ing. R. Urbansky

UNIVERSITY OF KAISERSLAUTERN

SIMO and MIMO Concepts for Fibre Optical Communications

M. Siegrist, A. Dittrich, W. Sauer-Greff, R. Urbansky
TU Kaiserslautern, Institute of Communications Engineering

6. Session
ITG-Fachgruppe "Angewandte Informationstheorie"
7. Oktober 2005, Kaiserslautern

Overview

- Nonlinear State-Based Transmission Systems
- Optical Transmission System considered as
  - SIMO System
  - MIMO System
- Simulation Results

UNIVERSITY OF KAISERSLAUTERN

ITG Fachgruppe “Angewandte Informationstheorie”
Kaiserslautern, 07.10.2005

Sender- und Empfängerstrukturen für codierte MIMO-Übertragung

Carsten Bockelmann
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Department of Communications Engineering

Outline

- Motivation
- Space-Time Codes
  - Multistratum Space-Time Codes
  - Universal Space-Time Codes
- Results
  - Uncoded Transmission
  - Coded Transmission
- Conclusions

Motivation

- Advantages of MIMO systems
  - Diversity Gain
  - Higher Rates
  - Both for recent ST schemes

- Analysis of performance is in general based on uncoded transmission
- But existing systems (e.g. UMTS, WLAN) utilize channel coding

Coded MIMO systems should be evaluated to gain insight into the performance and feasibility of complex coded ST schemes.

Space-Time Codes

- Evaluated Space-Time Codes
  - V-BLAST (spatial multiplexing)
  - Multistratum Space-Time Codes (MSSTC)
  - Universal / Complex Field Space-Time Codes (USTC)

- General system model

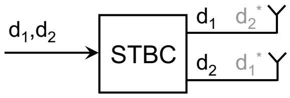
$$\underline{\mathbf{Y}} = \underline{\mathbf{H}} \cdot \underline{\mathbf{X}} + \underline{\mathbf{N}}$$

$\underline{\mathbf{Y}}$ Receive matrix
$\underline{\mathbf{X}}$ ST code word matrix
$\underline{\mathbf{H}}$ Channel matrix
$\underline{\mathbf{N}}$ Noise matrix

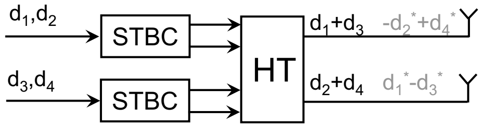
- Channel Model
  - N_T transmit antennas and N_R receive antennas
  - Quasi-static channel, constant for L channel uses
  - i.i.d Gaussian channel coefficients (flat fading)

Multistratum Space-Time Codes

Alamouti Space-Time Code
1 Symbol per channel use



2x2 MSSTC
2 Symbols per channel use

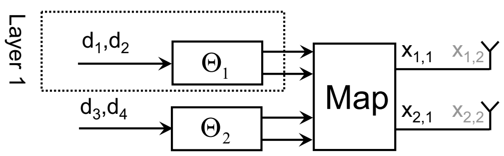


- Example of a quasi orthorgonal component code (4x4 antennas) which allows for max 4 strata
- Rotation θ ensures full diversity (optimized for symbol constellation)

$$\underline{\mathbf{X}} = \begin{pmatrix} \underline{d}_1 & -\underline{d}_2^* & -\underline{\tilde{d}}_3^* & \underline{\tilde{d}}_4 \\ \underline{d}_2 & \underline{d}_1^* & -\underline{\tilde{d}}_4^* & -\underline{\tilde{d}}_3 \\ \underline{\tilde{d}}_3 & -\underline{\tilde{d}}_4^* & \underline{d}_1^* & -\underline{d}_2 \\ \underline{\tilde{d}}_4 & \underline{\tilde{d}}_3^* & \underline{d}_2^* & \underline{d}_1 \end{pmatrix} \qquad \underline{\tilde{d}}_i = e^{j\theta}\underline{d}_i$$

Universal Space-Time Codes

2x2 USTC
2 Symbols per channel use



Layer Mapping

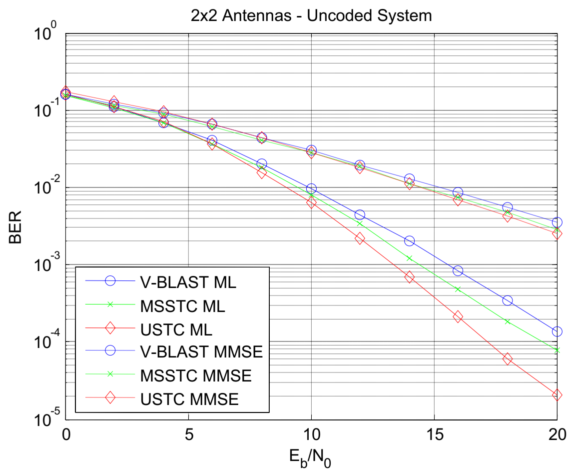
1	4	3	2
2	1	4	3
3	2	1	4
4	3	2	1

antenna ↑ time →

Code Matrix $\Theta_g = \beta^{g-1}\frac{1}{\sqrt{N_t}}\mathbf{F}_{N_t}^H \mathrm{diag}[1,\alpha,\cdots,\alpha^{N_t-1}]$ Where $\mathbf{F}_{N_t}^H$ FFT matrix

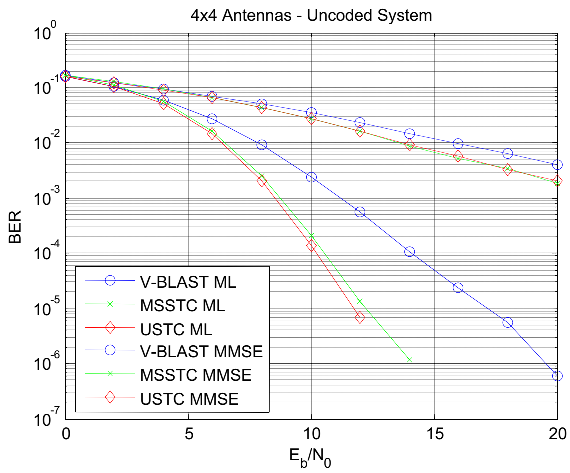
α,β are design parameters of the code, where β^{g-1} are the diophantine numbers

Uncoded Transmission



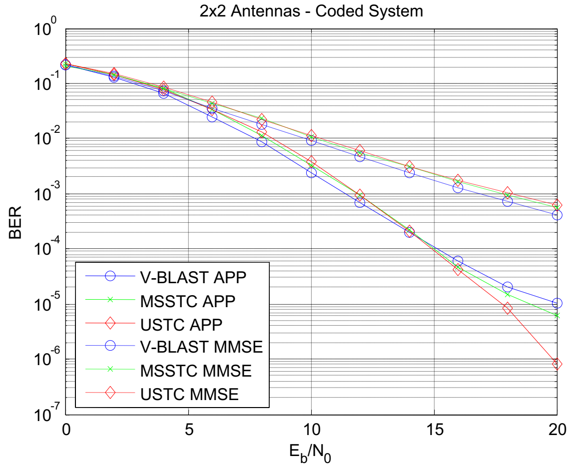
- MIMO Parameters
  - $N_T = 2$
  - $N_R = 2$
  - $L = 100$
- MSSTC
  - Orthogonal design
  - No full diversity

Uncoded Transmission



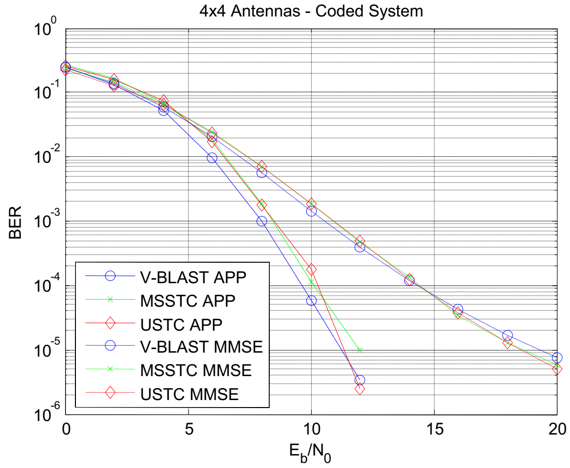
- MIMO Parameters
  - $N_T = 4$
  - $N_R = 4$
  - $L = 100$
- MSSTC
  - Quasi-orthogonal design
  - Full diversity

Coded Transmission



- MIMO Parameters
  - $N_T = 2$
  - $N_R = 2$
  - L = 100
- APP Detector
  - Max-APP
- Convolutional Code
  - $R_C = 0.5$
  - $L_C = 3$
  - Max-APP Decoder

Coded Transmission



- MIMO Parameters
  - $N_T = 4$
  - $N_R = 4$
  - L = 100
- APP Detector
  - List Sphere for MSSTC / USTC
- Convolutional Code
  - $R_C = 0.5$
  - $L_C = 3$
  - Max-APP Decoder

Conclusion

All codes are full-rate, but MSSTC and USTC gain transmit diversity

- uncoded MSSTC / USTC show better error performance
- coded MSSTC / USTC gain performance with respect to V-BLAST only at high E_b/N_0
- → Receiver complexity is wasted!

- Outlook
  - Adaptive transmission for MSSTC / USTC
    - SINR per Layer should be more stable
    - Less channel knowledge needed at transmitter
  - MIMO-OFDM for MSSTC / USTC